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ARSP–2 Volume I

GUIDANCE ON THE DEVELOPMENT OF WEAPON DANGER AREAS/ZONES

PROBABILISTIC METHODOLOGY – GENERAL PRINCIPLES

Edition B Version 1

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NATO LETTER OF PROMULGATION

19 November 2015

1. The enclosed Allied Range Safety Publication ARSP-2 Volume I, Edition B, Version 1, GUIDANCE ON THE DEVELOPMENT OF WEAPON DANGER AREAS/ZONES FOR USE BY NATO FORCES — PROBABILISTIC METHODOLOGY — GENERAL PRINICIPLES, which has been approved by the nations in the MCLSB, is promulgated herewith. The agreement of interested nations to use this publication is recorded in STANAG 2470.

2. ARSP-2 VOL I, Edition B, Version 1, is effective upon receipt and supersedes ARSP-2 VOL I, Edition A, Version 1, which shall be destroyed in accordance with the local procedure for the destruction of documents.

4 This publication shall be handled in accordance with C-M(2002)60.

Edvardas MAŽEIKIS
Major General, LTUAF
Director, NATO Standardization Office

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RECORD OF RESERVATIONS

CHAPTER	RECORD OF RESERVATIONS BY NATIONS
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RECORD OF SPECIFIC RESERVATIONS

[nation]	[detail of reservation]
DNK	Implementation will be reconsidered in the event that the present use of deterministic methods becomes stochastic.
POL	1. Poland reserves the right not to apply the provisions included in the STANAG, resulting from the current possibilities of armament systems, especially with regard to the use of compatible computer software and the related aspects of training security. 2. The postponing results from the need to develop a national response with regard to the STANAGs referenced in ARSP-2 Ed. B: 3606/ARSP-1, Vol. II, 2921, 2402.

Note: The reservations listed on this page include only those that were recorded at time of promulgation and may not be complete. Refer to the NATO Standardization Document Database for the complete list of existing reservations.

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CHAPTER 1
GENERAL

0101. INTRODUCTION

1. A Weapon Danger Area (WDA) is that area associated with firing a weapon where the risk of death or injury exceeds some threshold. The risk outside the WDA does not exceed this threshold and hence the risk to people outside the WDA is acceptable or tolerable. A Weapon Danger Zone (WDZ) extends this into three dimensions. Traditionally, WDA have been developed using deterministic methodology and WDA are extended into WDZ by using a constant height above the WDA. In both cases the level of risk associated with the area or zone has been assessed as acceptable or tolerable, but has not been explicitly quantified. In order to quantify the levels of risk we have to use a probabilistic methodology.

2. Weapon Danger Area/Zones (WDA/Z) encompass the ground and airspace for lateral and vertical containment of projectiles, fragments, debris, and components resulting from the firing, launching, and/or detonation of ordnance. WDA/Z account for weapon accuracy, failures, ricochets, and broaches/porpoising of a specific weapon/munition type. WDA/Z developed using deterministic principles, where a number of worst case assumptions are used, are usually bigger than they need to be, and this often results in the use of large areas and can constrain training. The probabilistic methodology can be used to handle more complex situations:

a. Specific range danger areas/zones (RDA/Z) to account, for example, for local terrain and local met conditions can be developed;

a. The methodology can applied to the specific situation using realistic data so that range space can be optimized;

b. Ranges can be designed to contain projectiles;

c. Probabilistic analyses can provide information that can be used for other risk management purposes, for example to quantify the risk to range infrastructure;

d. Probabilistic analyses can provide information that can be used to diagnose problems for conceptual or existing ranges.

3. It is important to note that the use of a probabilistic methodology is not a universal remedy. Whilst is has many advantages over deterministic methodology, probabilistic models have to be developed and data needs to be gathered and analyzed for use in these models and this may not be a simple process.

0102. AIM

1. Whereas WDA are traditionally classified by weapon type and role, the general principles described here apply to all weapon systems and no such distinctions need to be introduced. This aim of this document is to describe these general principles so that they may be applied for all weapon systems and appropriate WDA/Z may be developed.

0103. SCOPE

1. This publication is relevant to the development of WDA/Z for all weapon systems. Although the description here is based on ballistic weapons the principles apply to all weapon systems and lasers (either used as part of a system or as weapons). Specific information for various categories of weapon systems is given in related publications.

0104. VOCABULARY AND ABBREVIATIONS

2. A list of terms and abbreviations used in this publication are provided in the Lexicon.

0105. RELATED DOCUMENTS

1. This is one of a sequence of Allied Range Safety Publications (ARSPs) that are concerned with the development of WDA/Z for a variety of weapon systems for use by NATO forces in a variety of roles. The framework is shown in Figure 1.1. Brief descriptions of each ARSP are given below:

a. Volumes in STANAG 2401 (Reference 1) with ARSP-1 cover the deterministic methodology:

b. Volume I (Reference 2) contains a description of the factors that are relevant to the use of unguided weapons.

(1) Volume II (Reference 3) contains a description of the application of the factors from Volume I, and provides generic danger area outlines together with nation dependent numerical values for the factors.

b. Volumes in STANAG 2470 (Reference 4) with ARSP-2 cover the probabilistic methodology:

(1) Volume II (Reference 5) contains a description of the application of these principles to unguided weapons. It includes descriptions, and in some cases detailed specifications, of the models that may be used when applying the probabilistic methodology to the factors in ARSP-1 Volume I.

(2) Volume III (Reference 6) contains a description of the application of these principles to guided weapons (GW).

(3) Volume IV contains a description of the application of these principles to unmanned aerial vehicles (UAVs). This is an update of STANAG 2402, Edition 2 (Reference 7).

c. Volumes in ARSP-3 cover the acquisition and analysis of data for use with both deterministic and probabilistic methodologies:

(1) Volume I contains a description of trials procedures and data analysis for aimer deviations.

(2) Volume II contains a description of trials procedures and data analysis for free-flight data.

(3) Volume III contains a description of trials procedures and data analysis for fragmentation data.

(4) Volume IV contains a description of trials procedures and data analysis for impact and post-impact models.

d. STANAG 3606 (Reference 8) with ARSP-4 (Reference 9) covers the factors relevant to lasers and the application of deterministic and probabilistic methods to lasers.

Figure 1.1 — Framework of Allied Range Safety Publications.

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CHAPTER 2
RISK

0201. INTRODUCTION

1. The determination of a WDA/Z and its use as an exclusion area/zone is part of a risk management process that aims to ensure that the risk to the general public, civilian, participating and/or non-participating military personnel arising from the use of weapon systems is kept to an acceptable or tolerable level.

2. A framework for risk management (References 10 and 11) is shown in Figure 2.1. This shows the place of risk analysis, which consists of hazard identification and risk estimation, within the overall framework. In this publication we are only directly concerned with the two risk analysis stages.

3. Concerns over the levels of risk that are considered acceptable or tolerable, which are relevant in the risk evaluation stage, are dealt with as national issues and are prescribed in national documents.

4. In the hazard identification stage the aim is to identify all the hazards associated with the use of a particular weapon system. In many cases the hazards are generic to particular classes of weapon system and those relevant are identified in standard lists. These generic lists should be developed, documented, and maintained. Where particular weapon systems do not conform to a general class and/or present additional hazards a specific document may be produced for each individual weapon system.

5. An example of a generic list is ARSP-1 Volume I (Reference 1), which provides a list of factors (i.e. hazards) that should be considered when developing WDA/Z for unguided weapon systems. This was produced, and is maintained, as a separate document from the document that provides the WDA/Z. For other generic weapon systems the list may be included in the same document. Guided weapon systems fit into the second category as each guided weapon system tends to have unique guidance and aerodynamic behaviour. Irrespective of the location of the list it is important to realize that the hazard list is the same for both deterministic and probabilistic methodologies.

6. It is in the risk estimation stage that the difference between deterministic and probabilistic methodologies is seen. In the probabilistic methodology a direct estimate of the risk is attempted. With the deterministic methodology worst case assumptions may be used to bound the risk without actually estimating it directly. In addition heuristic methods that appear to provide an appropriate level of risk are used.

0202. PROBABILITY, FREQUENCY, AND UNITS

1. Before describing some common measures of risk some of the terminology of risk is described. There are many different terminologies used for risk and it is often difficult to get agreement on the terminology that should be used in a particular situation. Two examples of the definition of risk are:

   a. Risk is the probability of a particular adverse consequence (Reference 12);

   b. Risk is a combination of frequency and adverse consequence (Reference 13).

2. Problems arise when one definition is provided and an alternative is used in calculations. Similar confusion arises between the terms hazard and risk. It is important to use an appropriate definition and ensure that what is calculated corresponds to the definition. Much of this confusion can be eliminated by (1) understanding the difference between probability and frequency, and (2) always using units when quoting probabilities or frequencies, i.e. not quoting numbers such as 1 in 1 000 000 on their own.

3. Probability is defined as “a real number in the scale 0 to 1 attached to a random event” (Reference 14). It can be related to a long-term relative frequency of occurrence or to a degree of belief that an event will occur — both approaches lead to much the same principles — but the relative

frequency interpretation is the most common. Frequencies occur as a result of the calculation of expectations, which combine probabilities per initiating event and the number of initiating events. A key difference between probability and expectation is that probabilities are constrained to the scale 0 to 1 whilst frequencies are not, and hence any quantity that can be greater than 1 is a frequency.

4. A probability (or frequency) is a probability (or frequency) of something and there are some units involved, e.g. 1 death in 1 000 000 person years. All the various components of a calculation have their own units and both the numbers and units are combined appropriately. Often the units are ignored, when their use would indicate that something is wrong with the calculation. Sometimes the result is a risk but does not match the definition used, and sometimes the result is not a risk at all.

5. As an example of the use, and combination, of units consider the calculation for a probability of death per round fired. This could arise as a result of the use of a weapon system firing a single projectile and the calculation could combine two terms as:

$$\begin{array}{ccccc} P_{D/R} & = & P_{D/H} & \times & P_{H/R} \\ \dfrac{\text{deaths}}{\text{round}} & = & \dfrac{\text{deaths}}{\text{hit}} & \times & \dfrac{\text{hits}}{\text{round}} \end{array} \qquad (2.1)$$

where $P_{D|H}$ may depend on the part of the body hit and the presence, or otherwise, of body armour;

and $P_{H|R}$ is calculated assuming people are present the whole time the weapon system is in use.

0203. SOME COMMON MEASURES OF RISK FOR WDA/Z

1. Here we describe some measures of risk that are in common use for developing WDA/Z. The simplest is probability of escape. The most general is frequency of death from which, a range of similar measures can be derived.

2. A list of measures together with options for calculating hazard and risk (from Chapter 6) is provided in Annex D and with options for the development of WDA from density functions (from Chapter 7) is provided in Annex E.

3. Probability, or frequency, of escapes from the WDA/Z in escapes per firing.

   a. Traditionally this has been used with weapon systems for developing WDA/Z. When we assume that the weapon system fires a single projectile that remains in one piece and a single projectile does not kill more than one person the result is a probability. Where multiple projectiles result from a single firing, e.g. for projectiles that break up on impact with the terrain, or projectiles that fragment, the result is a frequency as a single firing can result in multiple escapes.

   b. This is a hazard (i.e. not a risk) as the escape from the WDA/Z is not necessarily an adverse consequence. However, it can be converted to a risk by adding terms, which are then assumed to have value 1. For example, if we add two terms for hits per escape and deaths per hit we obtain:

$$\begin{array}{ccccccc} P_{D/R} & = & P_{D/H} & \times & P_{H/E} & \times & P_{E/R} \\ \dfrac{\text{deaths}}{\text{round}} & = & \dfrac{\text{deaths}}{\text{hit}} & \times & \dfrac{\text{hits}}{\text{escape}} & \times & \dfrac{\text{escapes}}{\text{round}}. \end{array} \qquad (2.2)$$

   c. When worst case values of 1 are taken for the additional terms it is seen that this places a bound on the risk of death in terms of deaths per round:

$$\begin{array}{ccccccc} P_{D/R} & \leq & 1 & \times & 1 & \times & P_{E/R} \\ \frac{\text{deaths}}{\text{round}} & = & \frac{\text{deaths}}{\text{hit}} & \times & \frac{\text{hits}}{\text{escape}} & \times & \frac{\text{escapes}}{\text{round}} \end{array}. \quad (2.3)$$

d. Probability of escape and any measures of risk derived from it are properties of the WDA/Z. The levels of hazard or risk apply to the exterior region between the WDA/Z boundary and the boundary of the zero energy area/zone. They are only indirectly functions of position — generally the risk decreases from the boundary of the WDA/Z to zero at the boundary of the zero energy area/zone but the calculation of probability of escape cannot demonstrate this.

4. Frequency of death in deaths per person year, known as annual individual risk of death (IR) (Reference 15).

a. This is the most common measure of risk of death used in compiling national statistics, where it is calculated on an actuarial basis as the ratio of deaths from a particular activity in a year divided by the number of people that participated in that activity in that year.

b. It is the accepted measure of risk within GBR for accidental death whilst at work and is the preferred measure used by GBR Government Departments for setting safety standards (References 16 and 17) and is being adopted by some other nations for other defence related purposes (Reference 18).

c. The calculation for the frequency of deaths per person year is derived from (2.1) by adding two terms:

$$\begin{array}{ccccccccc} F_{D/PY} & = & F_{R/Y} & \times & P_{D/H} & \times & P_{H/R} & \times & E_{Y/PY} \\ \frac{\text{deaths}}{\text{person}\cdot\text{year}} & = & \frac{\text{rounds}}{\text{year}} & \times & \frac{\text{deaths}}{\text{hit}} & \times & \frac{\text{hits}}{\text{round}} & \times & \frac{\text{years}}{\text{person}\cdot\text{year}} \end{array} \quad (2.4)$$

where the additional term $F_{R|Y}$ is the frequency, i.e. number, of rounds fired in a year, and $E_{Y|P \cdot Y}$ is the exposure i.e. the proportion of time that people are present when the weapon system is in use.

d. Although it is a frequency, as the calculation can produce numbers greater than 1, it is often referred to as a probability because any activity that produced numbers greater than 1 would obviously be unacceptable.

5. Individual risk of death (IR) can also be used on a per event basis.

e. The calculation for the frequency of deaths per event is derived from (2.4) by considering events instead of years and removing the exposure term:

$$\begin{array}{ccccccccc} F_{D/PE} & = & F_{R/E} & \times & P_{D/H} & \times & P_{H/R} & \times & E \\ \frac{\text{deaths}}{\text{person}\cdot\text{event}} & = & \frac{\text{rounds}}{\text{event}} & \times & \frac{\text{deaths}}{\text{hit}} & \times & \frac{\text{hits}}{\text{round}} & \times & \frac{1}{\text{person}} \end{array} \quad (2.5)$$

6. Individual risk is a function of position and it varies widely over the WDA/Z — it is high directly in front of the firer and generally decreases to zero at the boundary of the zero energy area/zone, and the calculation of individual risk will determine this. In most cases IR is zero before the boundary of the zero energy area/zone because projectiles near the boundary generally have insufficient energy to cause death.

7. One advantage of the use of individual risk over the use of probability of escape is that the examination of the different terms in the calculation provides a direct link with risk mitigation measures. If the overall risk is too high, any combination of reduction in each of the four terms, in (2.4), can be

investigated. This already happens within most range management systems but the link with the risk estimation process is not evident:

a. The use of the WDA/Z as a restricted zone, keeping people out of the area or zone, reduces the exposure term $E_{Y|P \cdot Y}$ to zero and the risk inside the WDA/Z is zero.

b. The use of body armour reduces the probability of death or injury given a hit term $P_{D|H}$.

c. The probability of hit term $P_{H|R}$ can be reduced by range design, by for example the addition of baffles or barriers, which stop projectiles entering specific locations within the original WDA/Z.

d. Finally, the frequency of firing $F_{R|Y}$ can simply be reduced, or limited, to a level that produces an appropriate overall level of risk.

8. Where IR is concerned with the risk to an individual, collective risk (CR) considers all people exposed to the hazard. It is measured in terms of the expected number of deaths, either per event or per year. For example, the annual collective risk of death is given by:

$$\begin{aligned} F_{D/Y} &= \sum_P F_{D/PY} \\ \frac{\text{deaths}}{\text{year}} &= \text{person} \times \frac{\text{deaths}}{\text{person} \cdot \text{year}} \end{aligned} \qquad (2.6)$$

9. It is also easy to derive other measures of risk from risk of death. We may decide to use frequency of injury, for example in injuries per person year, or frequency of death, for example in hits per event.

a. The first of these is derived from (2.4) by replacing deaths per hit by injuries per hit:

$$\begin{aligned} F_{D/PY} &= F_{R/Y} \times P_{D/H} \times P_{H/R} \times E_{Y/PY} \\ \frac{\text{injuries}}{\text{person} \cdot \text{year}} &= \frac{\text{rounds}}{\text{year}} \times \frac{\text{injuries}}{\text{hit}} \times \frac{\text{hits}}{\text{round}} \times \frac{\text{years}}{\text{person} \cdot \text{year}} \end{aligned} \qquad (2.7)$$

b. The second is derived from (2.5) by using the value of 1 for deaths per hit. This produces an upper bound on the individual risk of death and is considered a cautious approach that avoids the requirement to specify wounding models:

$$\begin{aligned} F_{D/PE} &\leq F_{R/E} \times 1 \times P_{H/R} \times E_{Y/PE} \\ \frac{\text{deaths}}{\text{person} \cdot \text{event}} &= \frac{\text{rounds}}{\text{event}} \times \frac{\text{deaths}}{\text{hit}} \times \frac{\text{hits}}{\text{round}} \times \frac{1}{\text{person}}. \end{aligned} \qquad (2.8)$$

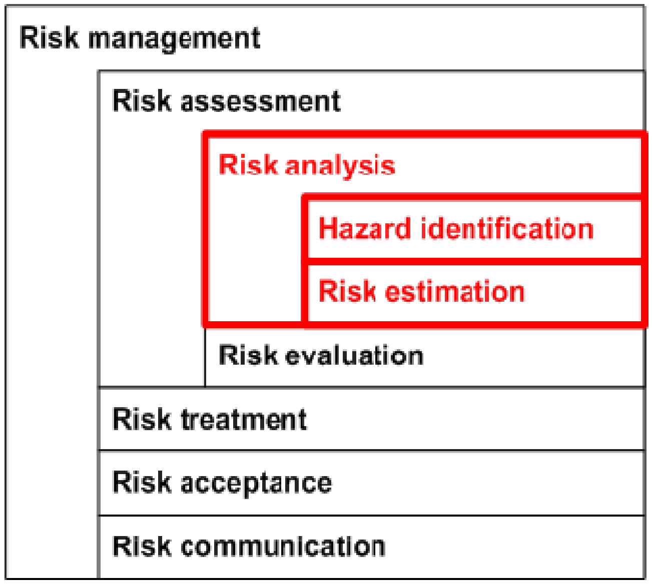

Figure 2.1 — Risk management framework (showing the place of risk analysis, which consists of hazard identification and risk estimation, within the overall framework)

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CHAPTER 3
ILLUSTRATIONS AND EXAMPLES OF THE PROBABILISTIC METHODOLOGY

0301. INTRODUCTION

1. A purely mathematical presentation of the principles of the probabilistic methodology is neither easy to understand nor necessary. The principles are illustrated here using examples involving unrealistically simple weapon systems before some of the factors that have to be considered in more complicated situations are described.

2. We start by considering only the calculation of WDAs for a single static firing position. A problem of unguided weapon free flight where the only random input is the elevation is described in clause 302. We show how a WDA can be developed for this problem before complicating factors are added one at a time and the effect on the development of a WDA is examined in clauses 303 to 305. Clause 306 contains some simple examples that involve complications, such as moving platforms and moving targets. The determination of WDA/Zs for guided weapons (GW) is covered in clause 307. Finally the determination of composite WDA/Zs, which may arise from the use of multiple firing positions and/or different weapon systems, is described in clause 308.

3. Whilst reading this chapter it may be useful to consider how WDA/Zs would be obtained using a deterministic methodology. In some cases the deterministic methodology is difficult to apply, in others the use of a purely probabilistic methodology is not possible and we end up with a hybrid methodology.

0302. ILLUSTRATION 1 — AN UNGUIDED WEAPON LAUNCHED WITH VARYING ELEVATION

1. The simplest case that can be considered is that of an unguided weapon where all parameters are constant apart from the elevation, which is randomly distributed about some mean value. The terrain is flat, the azimuth is zero, there is no wind, the trajectory is planar (i.e. there is no drift), and hence the impact location for a given elevation is given by the range from the firing position. The maximum elevation is below that needed for maximum range, each unique elevation corresponds to a unique impact position, and range increases as elevation increases.

2. The WDA is taken to be from the firing point to the range that is exceeded by only 1 in 1 000 000 rounds and the WDA then contains 999 999 in 1 000 000 rounds. Once we know the distribution of elevations we have to determine this range in order to develop the WDA.

3. In Figure 3.1 two trajectories T_1 and T_2 with ranges $R_1 < R_2$ corresponding to two elevations $E_1 < E_2$ are shown. As trajectories do not cross each other it is evident that all elevations between E_1 and E_2 produce trajectories between T_1 and T_2 with ranges between R_1 and R_2. If the probability that an elevation is greater than E_2 is P_2, this is also the probability that a trajectory is above T_2 and a range is greater than R_2. The WDA is found by finding the elevation that is exceeded in 1 in 1 000 000 rounds and then computing the range at this elevation.

0303. ILLUSTRATION 2 — AN UNGUIDED WEAPON LAUNCHED WITH VARYING ELEVATION AND VARYING AZIMUTH

1. A simple modification, a randomly varying azimuth, is now made to illustration 1 that introduces a significant complication. This azimuth is randomly distributed about a mean value of zero. For a unique elevation and azimuth the impact location is again unique but is now specified by two coordinates:

$$\begin{aligned} x &= R\cos A \\ z &= R\sin A \end{aligned} \qquad (3.1)$$

where A is the azimuth and the range R depends on elevation as before.

2. In Figure 3.2 a number of trajectories with impact locations corresponding to different elevations and azimuths are shown. As before the trajectories do not cross and the relationship between ranges and elevations is the same i.e. all elevations between E_1 and E_2 produce ranges between R_1 and R_2.The relationship between azimuth and cross-range position z is similar — for a given range, all elevations between A_1 and A_2 produce cross-range positions between $z_1 = R\cos A_1$ and $z_2 = R\cos A_2$.

3. In trying to develop a WDA we now have a more complicated problem. If we specify a probability of escape, such as 1 in 1 000 000 firings, how are we to choose the area that corresponds to this? This question occurs because there is now no unique area that corresponds to a given probability. There are a number of ways to overcome this:

a. Treat range and azimuth independently and choose a probability for each such that the combination is 1 in 1 000 000.

b. This is not unique as we could choose to use maximum range (thus covering all ranges) and azimuth limits to contain 999 999 in 1 000 000 firings. This is still not unique as the azimuth limits are not determined and a further apparently arbitrary choice has to be made.

c. The usual way of choosing is to use a limit on range that covers $\sqrt{0.999\,999}$ and limits on azimuth that cover the same proportion i.e. $\sqrt{0.999\,999}$. The combined area then covers the required proportion. As with the previous choice this is still not unique as the azimuth limits are not determined.

d. The two common methods of choosing the azimuth limits are to (a) use lower and upper limits such that the probability of being outside either is the same, and (b) choose lower and upper limits such that the probability density has the same value at both limits. Method (a) corresponds to the classical method for choosing central confidence intervals (Reference 19), whilst (b) corresponds to that used in Bayesian methods (Reference 20). Note that for a symmetric probability density (a) and (b) are equivalent.

e. Use a contour in the probability function and join it back to the launch point. This is equivalent to a two dimensional application as in a. (3) above. It has the property that the area chosen is a minimum.

0304. ILLUSTRATION 3 — AN UNGUIDED WEAPON WITH A FRAGMENTING WARHEAD

1. We complicate illustration 1 by adding a simple fragmenting warhead that is assumed to function on impact and has a constant circular fragmentation pattern.

2. The usual method for dealing with fragmenting warheads is to add a burst safety distance (BSD) to the existing WDA. It is often assumed that the original WDA and the new combined WDA and BSD have the same frequency associated with them. That this is not the case as can be seen by examination of the following example.

3. Assume that we have chosen the original WDA to cover all but 1 in 1 000 000 firings and use a BSD that encloses all fragments from the warhead event. The resulting combined WDA and BSD is shown in Figure 3.3 together with a single BSD for another warhead event. Even though this additional warhead event occurs with probability below 1 in 1 000 000 when the warhead produces a large number of fragments it is now possible to have the frequency of escape being higher than the original 1 in 1 000 000 firings.

4. Only the probabilistic methodology can be used to develop a correct WDA for this case. The distribution of the final resting places of fragments has to be constructed and the WDA chosen to contain all but 1 fragment in 1 000 000 firings of the weapon.

0305. ILLUSTRATION 4 — AN UNGUIDED WEAPON WITH RICOCHET

1. We now complicate illustration 2 by adding ricochet with the terrain on impact. Now ricochets may occur, depending on the impact conditions. Example trajectories including first ricochets after impact are illustrated in Figure 3.4.

2. As the ricochet process is random there is no apparent pattern to the locations of the final resting places. Where a complication arises in illustration 1 when low and high angle fire are present together because the launch conditions cannot be determined from the impact conditions, here the situation is much more complicated. There is no ordered relationship between the initial elevation and the final down-range position or between the initial azimuth and final cross-range position. It would appear that any final resting place can be reached in an infinite number of ways from any combination of the initial angles.

3. In the deterministic methodology, opening angles to allow for ricochet are applied from the boundaries of the region containing the initial impact area. As with the use of a combined WDA and BSD there is no guarantee that the probability or frequency associated with the resulting WDA has any particular relationship to that of the original WDA.

4. As with illustration 3, only the probabilistic methodology can be used to develop a WDA for this case. The distribution of the final resting places of projectile has to be constructed and the WDA chosen to contain all but 1 projectile in 1 000 000 firings of the weapon.

0306. FURTHER EXAMPLES

1. The four illustrations above have shown how the probabilistic methodology is adapted to handle simple complications. Even with these simple problems it becomes clear that the application of the probabilistic method is not straightforward. In this clause we briefly describe some other examples with additional complications that illustrate a number of things — the probabilistic data that is required to use the methodology — the differences between the deterministic and probabilistic methodologies — and problems where the use of a purely probabilistic methodology is difficult and the use of a deterministic method is easier.

2. The first example involves the use of explosives in training, which can involve training for operational Explosive Ordnance Disposal (EOD) or training in use of demolition charges, where we wish to determine the WDA/Z. Some of the additional factors that have to be considered are:

a. The ammunition or explosive is deliberately initiated.

b. For EOD this will almost certainly involve non-design mode functioning of the ammunition on, or in, the ground. How does the ammunition function? We usually only have data for design mode functioning in free air.

c. For EOD the position and orientation of the ammunition may be unusual. The burst safety distances that are used in other circumstances are determined for the orientations and velocities that arise in standard firing scenarios and here we have to use the data in situations where it may not be valid.

d. The outcome may depend on other weapons/mitigations that are involved. For demolition devices the break-up of the structure may produce fragments that travel further than those from the device.

3. The next example involves the use of a laser, say a range finder, or target designator, where we wish to determine the laser danger area/zone. The development of a danger area for a laser is similar to that for a ballistic weapon, except that it involves eye damage rather than death or injury caused by being hit by projectiles. Some of the factors considered are:

a. The laser parameters — analogous to the aerodynamic parameters of a projectile.

b. The pointing error for the laser system — analogous to the aimer deviations for a ballistic weapon system.

c. The probability of a person being irradiated — analogous to being hit by a projectile.

4. Some of the additional factors that have to be considered are:

a. The probability of an irradiated person looking at the laser.

b. The probability that atmospheric effects, such as scintillation, increase the radiant exposure entering the eye.

c. The probability of the received exposure causing ocular damage.

5. The next example involves a fixed wing aircraft flying over a range, with nominal altitude, run-in heading, dive angle, and airspeed. The weapon system is fired at a fixed point target, and we wish to determine the WDA/Z either over time or as a function of release envelope. Some of the additional factors that have to be considered are:

a. The altitude, run-in heading, dive angle, and airspeed are all subject to error and their distributions have to be specified.

b. How is the weapon system aimed? Is it manual or is a targeting system used? This will determine whether the aimer error depends on the pilot or is a function of the fire-control system.

c. Once we can develop WDA/Z for known firing situations, how do we apply these? Do we want a single composite WDA/Z that covers all scenarios that could occur, or a sequence of WDA/Z that change with time and firing situation?

6. The next example involves a naval platform engaged in gun fire support, firing from ship to shore, and we wish to determine the WDA/Z for a target area. Some of the additional factors that have to be considered are:

a. The target is an area and not just a point. How do we deal with the distribution of aiming points within this target area? Do we specify a distribution for these?

b. The weapon system will involve a number of electronic systems on board the platform. How does the interaction of these contribute to the aimer error? A model for aimer error may be difficult to derive.

c. The movement of the platform is determined by the sea state, which is beyond the control of the platform and this will contribute to the aimer error. Do we have to determine the WDA/Z as a function of sea state?

7. The next example involves a helicopter, hovering or manoeuvring over a range, firing a machine gun out of the side door at a set of fixed targets, and we wish to determine the WDA/Z. Some of the additional factors that have to be considered are:

a. The weapon system is unstable. What additional allowance do we make for aimer error?

b. The projectiles experience wind shear as they emerge from the boundary layer surrounding the helicopter. What are the actual meteorological conditions and what effect do these have?

c. What effect does the downwash from the rotor have? With some projectiles a proportion of those fired is thrown of course by the downwash. How do we allow for this?

8. The next example involves the determination of a “firing box”. A weapon system is being fired at a known point target and the safety boundary is known — it could for instance be the boundary of a

range. We wish to determine those firing positions that we can use without the risk being inappropriate at the safety boundary.

a. This requires the development of a WDA/Z for all valid firing positions and keeping a list of those where the WDA/Z results in the risk being appropriate.

b. The checking of the infinite number of firing positions is not practical and one approach is to use a grid over the area of interest. With an assumption that the relationship between the risk levels from the resulting WDA/Z are continuous this marks out the firing box but it should be noted that the "box" may not be a regular shape or indeed a single area or zone.

9. A similar example is that of determining a "target box". A weapon system is being fired from a known firing position and the safety boundary is known. We wish to determine the target positions that we can use without the risk of projectiles escaping outside the safety boundary. The same procedure used in the previous example can be adopted with the result being again a "box" that may not be a regular shape or indeed a single area or zone.

0307. GUIDED WEAPONS

1. The distribution of the trajectories and subsequent final resting places for unguided weapons are subject to disturbances that are principally continuous. For guided weapons on the other hand many of the disturbances to which they are subject are discrete. These arise because of individual faults, for example in the guidance system, that change the nature of the trajectories.

2. In order to analyse the behaviour and determine the trajectories and any associated probabilities from them, it is necessary to handle the outcome of each combination of discrete faults. The distributions for each outcome are then combined with frequencies corresponding to their probability of occurrence obtained from the underlying fault tree analysis.

3. This situation is beyond the deterministic methodology and hence a probabilistic approach is used. The probabilistic methodology used for guided weapons is identical to that described in this publication e.g. the use of simulation to obtain an approximation to the probability distribution of the final resting places.

0308. COMPOSITE WDA/Z

1. The examples listed above are all for a single firing position and a single target position. The term composite is used to refer to situations with multiple firing positions and/or multiple target positions and is used to indicate that the WDA/Zs are made up of a number of separate parts.

2. With the deterministic methodology a composite WDA/Z is usually developed by merely overlaying the individual WDA/Z for each firing and/or target position and this same approach can be used with the probabilistic methodology. However, it is important to realize that the risk for the composite WDA/Z is not the same as that for the individual WDA/Zs, though of course it may still be at an acceptable or tolerable level.

3. As an example, consider the use of a single firing position with two target positions illustrated in Figure 3.5. If the risks for the exposed population from firing at target position 1 and target position 2 are $R_1(E)$ and $R_2(E)$ individually, what is the risk from firing at the two target positions together? The answer depends on the measure of risk (individual risk or probability of escape) being used and exact meaning of the word together (at the same instantaneous time, or mutually exclusively).

4. Where individual risk is used and $R_1 = F_1$ and $R_2 = F_2$ are estimated from the use of N_1 and N_2 rounds per year and the two targets are used at the same instantaneous time the individual risk is

$$F_{12}(E) = F_1(E) + F_2(E) - S(N_1, N_2, \Delta t) \cdot F_1(E) \cdot F_2(E). \quad (3.2)$$

5. The last term, which represents the risk from being at risk by firing at target position 1 and target position 2 at the same time is the product of the two frequencies scaled to account for the fact that F_1 and F_2 are annual frequencies.

6. The scale factor $S\left(N_1, N_2, \Delta t\right)$ depends on the numbers of rounds fired and the time interval we consider — note that without the scale factor the product represents the frequency with which someone is hit by rounds fired at target 1 and target 2 in the same year.

7. At locations near the target positions this term could be significant, but at most locations outside the individual WDA/Z where the frequencies are small this term is negligible.

a. Where individual risk is used and $R_1 = F_1$ and $R_2 = F_2$ are estimated from the use of N rounds per year and the two targets are used mutually exclusively with these rounds split between the two target positions, i.e. $N_1 + N_2 = N$ the individual risk is

$$F\left(E\right) = \frac{N_1}{N} F_1\left(E\right) + \frac{N_2}{N} F_2\left(E\right) \le \max\left(F_1\left(E\right), F_2\left(E\right)\right). \tag{3.3}$$

b. Where probability of escape is used it does not matter whether the two targets are used at the same instantaneous time or mutually exclusively. The probability of escape is the sum of the expected number of escapes from firing at each target divided by the number of rounds fired. If the number of rounds fired at targets 1 and 2 in the ratio $N_1 : N_2$ the probability of escape is

$$P_{12} \le \frac{N_1 P_1 + N_2 P_2}{N_1 + N_2} \tag{3.4}$$

8. It is evident that the development of composite WDA/Z using overlays of individual WDA/Z is not necessarily a simple process even for the case of a single firing position and two target positions. For more complex cases, such as that of where sets of firing positions at different ranges are in use with a single set of target positions the calculation of risk using algebra is not feasible. However, with the probabilistic methodology the probabilities or frequencies can be combined using a computer program and WDA/Z can be developed for these cases in exactly the same way that they are developed for the simple cases.

9. The probabilistic methodology can also handle problems where different weapon systems, for example small arms and medium calibre systems, are used at the same time. A more interesting example is that for a weapon system where a laser is used with a ballistic system. With the deterministic methodology individual WDA/Z are used for the laser and the ballistic system are simply overlaid whereas the probabilistic methodology could be used to develop a composite WDA/Z for the laser and ballistic system considered together. This would require some common criterion for risk to be used as the combination of the use of probability of escape for a laser pulse and individual risk of death for the ballistic system would not make sense.

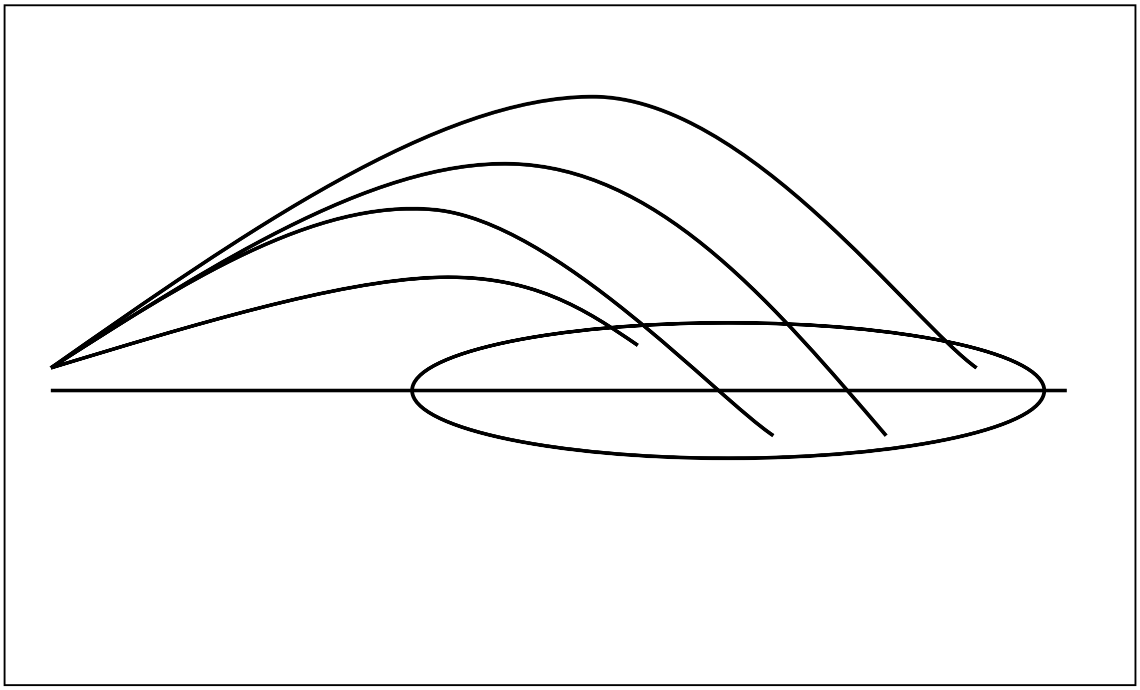

Figure 3.1 — Trajectories and impact ranges corresponding to two elevations for Example 1.

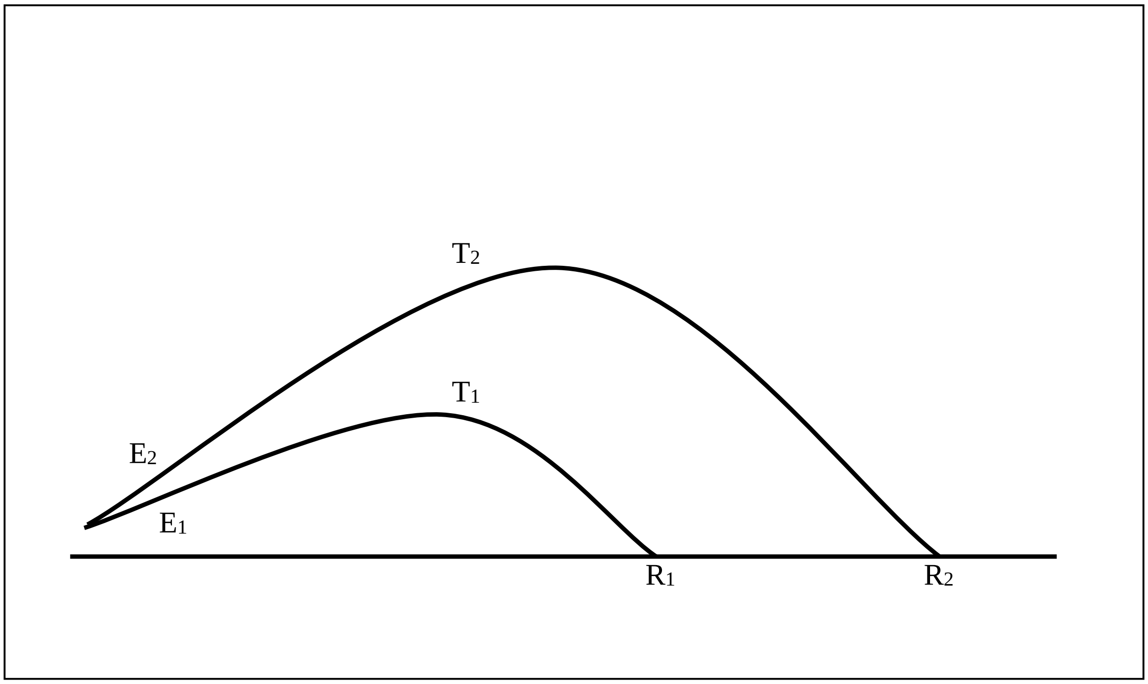

Figure 3.2 — Trajectories showing impact locations corresponding to different elevations and azimuths for Example 2.

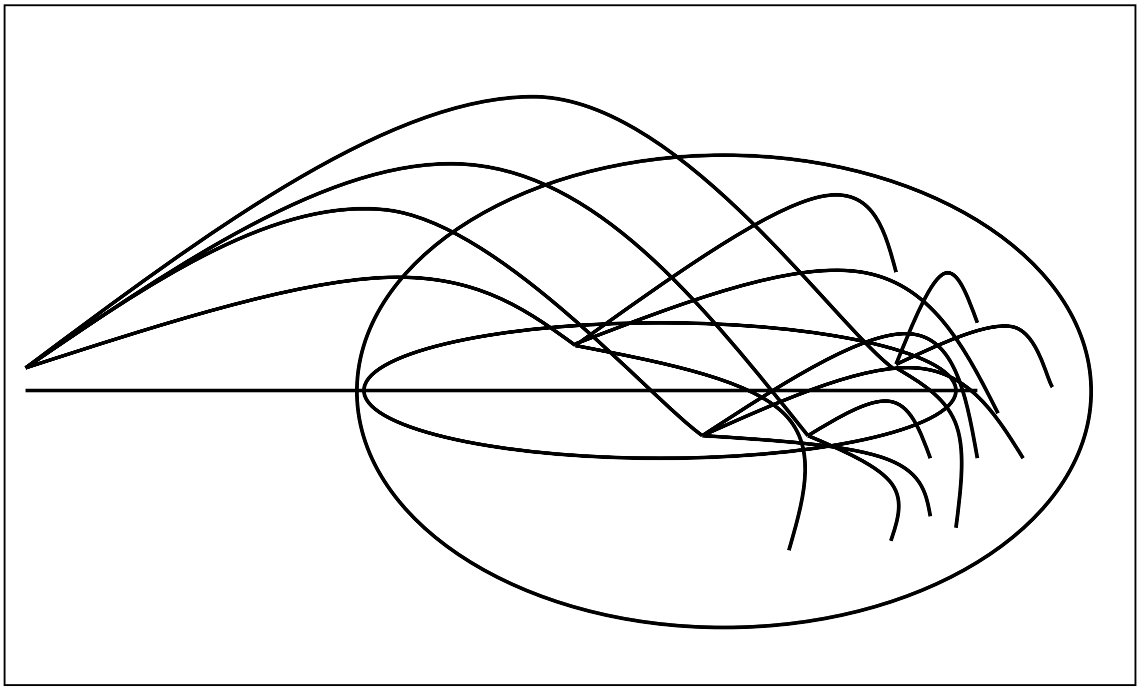

Figure 3.3 — Combined WDA and BSD with a single BSD outside for Example 3

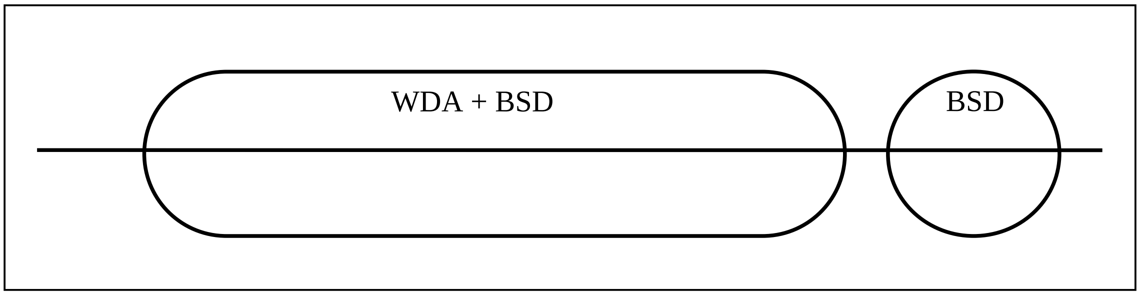

Figure 3.4 — Trajectories showing impact locations, and subsequent ricochets, corresponding to different elevations and azimuths for Example 4

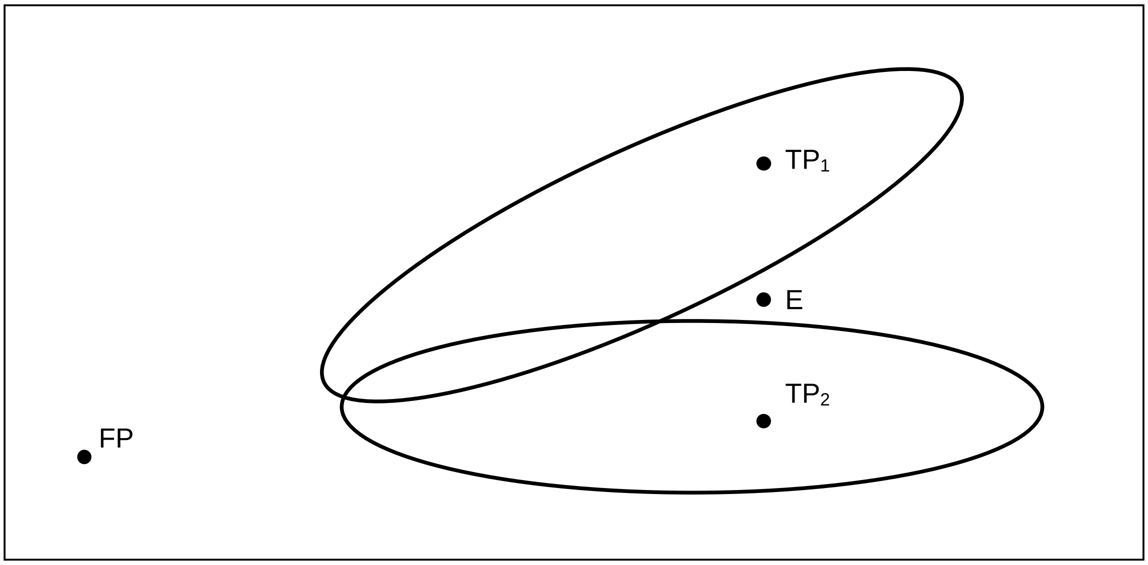

Figure 3.5 — WDAs for a single firing position (FP) and two target positions (TP_1 and TP_2) with an exposed population (E).

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CHAPTER 4
GENERAL PRINCIPLES OF THE PROBABILISTIC METHODOLOGY

0401. INTRODUCTION

1. After the illustrations given in Chapter 3 a more formal presentation of the principles of the probabilistic methodology is given here. We start by describing briefly the deterministic methodology before describing the probabilistic methodology. For practical applications the methodology used is rarely a pure implementation of the probabilistic method, because the development of probabilistic models and the acquisition and analysis of data for all events is often not possible, or indeed feasible. An important observation of the use of a hybrid methodology where some deterministic components and some probabilistic ones are used together is given.

0402. DETERMINISTIC, PROBABILISTIC, AND HYBRID METHODOLOGIES

1. With the deterministic methodology a sequence of “worst case” components is used to model the complete problem. It is important to recognize that these are worst in the sense that they yield the most severe solution to the complete problem and may not necessarily be the worst for the individual component. One obvious example of this is the launch elevation for an unguided projectile — if the elevations include that for maximum range use of the minimum and maximum angles will not reproduce the maximum range.

2. With the probabilistic methodology all random components are described in terms of probability distributions and these are used to construct a probability or frequency distribution for the complete problem. Practical methods for calculating this distribution are described in Chapter 5, whilst some theoretical methods are described in Annex A.

3. In hybrid models a mix of the two methodologies is used. Some components are treated probabilistically and some are handled deterministically. As with the pure deterministic methodology worst cases are used for the deterministic components — in the sense that they yield the most severe probabilistic solution to the complete problem.

0403. SENSITIVITY, VARIABILITY, AND UNCERTAINTY

1. When using any mathematical model that contains components where the parameters that describe the model of the component are not certain it is important to understand the sensitivity of the model to the various inputs. A sensitivity analysis estimates the rate of change of the output to changes in each of the inputs.

2. Variability — formally called aleatory uncertainty — arises because variables are random and individual values vary about the expected value. Variability may be characterized by the variance of the distribution of a variable and can be illustrated by calculating fractiles of the distribution.

3. Uncertainty — formally called epistemic uncertainty — arises because we have incomplete knowledge, either because our models are not adequate or because we do not know the true distributions or parameter values.

4. In the unlikely event that there are no uncertainties in the models from the parameters, there are no uncertainties in the complete model. This situation is characterized by variability on its own. If variability is present in a system we have to tolerate it as the only way to remove it is to redesign the system.

5. Where uncertainties are present in the models for the components because parameter values are unknown, and sample statistics are used in their place, uncertainty can be dealt with by specifying confidence intervals or limits for the quantities of interest. Techniques for calculating these intervals or limits for univariate quantities are available and are widely used in statistical methods. The uncertainty in these situations and the size of these intervals can be reduced by gathering more data.

6. In cases where there is no information about parameters in models the methodology described in clause 308 for composite WDA/Z can be adapted to deal with this. This methodology of combining results from different calculations applies in more general situations and can be used to obtain bounds in situations where there are several unknown parameters. In the example the probability of escape was

$$P_{12} \leq \frac{N_1 P_1 + N_2 P_2}{N_1 + N_2}. \quad (4.1)$$

a. Where N_1 and N_2 are not known an upper bound is obtained by noting that

$$P_{12} \leq \max(P_1, P_2). \quad (4.2)$$

b. Both variability and uncertainty are linked to sensitivity. The variability (or uncertainty) of a complete model is a product of the sensitivity of the complete model to a parameter and the variability (or uncertainty) in that parameter. It is a common mistake to calculate the sensitivity alone and rank the importance of the various parameters on the individual sensitivities even when there is no variability or uncertainty in some of the parameters.

c. Variability and uncertainty analysis is a useful tool in building complete probabilistic models as the results can be used to determine the inputs that are important and hence informing the priorities for gathering more information.

0404. THE DISTRIBUTION OF FINAL RESTING PLACES – "PROBABILITY OF ESCAPE"

1. Where WDA/Z are developed using the probability of escape criterion an estimate of the density function for the distribution of the final resting places is calculated. A contour corresponding to the specified acceptable probability of escape is found and is joined to the firing position using straight lines.

2. It is usually assumed that this method creates an upper bound for the risk outside the WDA/Z but it is important to note that it has to be assumed that no projectile that comes to rest inside the WDA/Z ever goes outside it. For projectiles with complex aerodynamic behaviour and/or in extreme meteorological conditions this assumption may be invalid.

3. For weapon systems where single projectiles are launched and these remain as single projectiles the resulting density function is a probability density function and integrates to 1. Hence the probability of escape is 1 minus the probability of containment and once either is known the other can be found by simple arithmetic.

4. For weapon systems where multiple projectiles arise from a single firing, for example munitions with fragmentation warheads, the resulting function is a density function as it integrates to more than 1. Hence the probability of escape and probability of containment both have to be calculated.

0405. INDIVIDUAL OR COLLECTIVE RISK — FREQUENCY OF DEATH OR INJURY

1. When using individual risk as a criterion for developing WDA/Z an estimate of the density function for the frequency of death (or serious injury) has to be calculated. This is more complicated than the calculation for the distribution of final resting places as it depends on other things:

a. The probability that a person at a particular location would be hit by a projectile;

b. The probability that a person hit by a projectile would be killed (or injured).

c. The exposure — the proportion of time that a person is present in a particular area whilst the weapon system is being used.

d. If the number of hits/injuries/deaths is to be calculated the population density at all locations is also required.

2. If there is only one person present the probability that a person at a particular location would be hit can be estimated at all locations in a single calculation. Once more than one person is present this is no longer the case as the people alter the calculation (e.g. a person being hit generally stops the projectile) and to obtain a correct result the people need to be part of the calculation.

3. There is a similar complication with the probability of kill (or injury) and again this has to be part of the calculation as the criterion for a kill depends on the mass and velocity of the projectile at the particular instant that the person is hit.

4. The exposure may depend on the nature of the people. The amount of time they spend in an area may depend on their occupation or we may be considering a transient population where for example people are crossing a range. The probability of a person being present at a particular location depends on the population density at the time the weapon system is being fired.

5. It is important to understand that exposure and the probability of being present are different. Where exposure is used the probabilities that are obtained apply to the area under consideration and not the people — so it does not matter how many people are present in the area. Where the probability of being present is being considered the expected number of deaths is directly proportional to the number of people present.

6. A correct calculation taking proper account of these factors is not practical in most cases. Fortunately, for the development of most WDA/Z, we can make some simplifying assumptions that do not introduce significant errors. Because there are not likely to be many people present the fact that the people alter the calculation of probability of hit can be ignored:

a. The probability of kill (or injury) is included in the calculation and we obtain the density distribution for people being killed (or injured), given that they are present the whole time, at all locations in a single calculation. This can be written

$$\mathrm{P}\left(\text{death}\middle|\text{exposed}\right)(x,z) \tag{4.3}$$

b. The probability of a person being exposed in a particular area can be written

$$\mathrm{P}\left(\text{exposed}\right)(x,z) \tag{4.4}$$

and is combined with (4.3) to obtain the density function for people being killed

$$\mathrm{P}\left(\text{death}\right)(x,z) = \mathrm{P}\left(\text{death}\middle|\text{exposed}\right)(x,z) \times \mathrm{P}\left(\text{exposed}\right)(x,z) \tag{4.5}$$

c. The probability of a person being present at a particular location can be handled in the same way to obtain

$$\mathrm{P}\left(\text{death}\right)(x,z) = \mathrm{P}\left(\text{death}\middle|\text{present}\right)(x,z) \times \mathrm{P}\left(\text{present}\right)(x,z) \tag{4.6}$$

0406. WEAPON DANGER ZONES — THREE AND FOUR DIMENSIONAL PROBLEMS

1. The previous clauses describe two dimensional problems for WDAs. A WDA can be extended into the third dimension to produce a WDZ. With the deterministic methodology this is usually achieved by specifying a constant height over the WDA. With the probabilistic methodology it is possible to produce a density function in three dimensions and derive a WDZ that varies in height over the WDA.

2. For moving targets a fourth dimension, time, is needed and a density function in three dimensions as a function of time is produced. It is important to note that the target has to be included in the calculation. As with people the target alters the calculation and here it is not possible to separate the two factors. To see this, consider a target that moves along the same trajectory as a projectile — when introducing the target after the calculation it is not possible to determine that it is hit only once by the projectile and the result obtained would be incorrect.

0407. COMPOSITE WDA/Z

1. An example regarding the development of a composite WDA/Z has been given in clause 308. There is little to add here except that the combination of probabilities or frequencies for the purpose of developing a composite WDA/Z is only valid when the probabilities or frequencies all have the same units. It is invalid, for example, to combine frequencies of death with probabilities of escape.

CHAPTER 5
PROBABILITY/FREQUENCY CALCULATIONS

0501. INTRODUCTION

1. There are many methods for handling the probability/frequency calculations. Analytic methods such as transformation of variables are feasible only for relatively simple situations, and some examples are described in Annex A. For the development of WDA/Z the only practical method available is simulation. Here Monte Carlo simulation and importance sampling are described. In situations where simulations are not feasible or required, data can be stored and interpolation used – this method is referred to as data cubes.

0502. PROBABILISTIC MODELS

1. In order to carry out the probability calculations each of the random components has to be specified by a probabilistic model, i.e. the complete specification of the joint probability density function has to be provided. A complete specification requires either (1) the formulae for the joint probability density function, or (2) the specification of one of the standard distributions together with values for its parameters.

2. As an example consider a specification of the distribution of the launch parameters (x, y, z, V, E, A) for a point-mass trajectory for an unguided projectile:

a. The launch position (x, z) is constant;

b. The launch height is taken to be normally distributed with mean $\mu = 1.7$ m, and standard deviation $\sigma = 0.05$ m;

c. The launch velocity is taken to be normally distributed with mean $\mu = 850.0$ m/s, and standard deviation $\sigma = 10.0$ m/s;

d. The launch elevation and azimuth are taken to be normally distributed with mean $(\mu_E, \mu_A) = (60 \text{ mils}, 0 \text{ mils})$, standard deviation $(\sigma_A, \sigma_E) = (0.1 \text{ mils}, 0.1 \text{ mils})$, and correlation $\rho_{EA} = 0.25$.

0503. MONTE CARLO SIMULATION

1. Monte Carlo simulation is a direct simulation of the problem being considered. In the context of WDA/Z it consists of the simulation of a large number of complete trajectories from launch through to coming to rest. In each trajectory, every input, such as the launch velocity and the probability of ricochet, is obtained by using random values generated from the appropriate distribution. The complete simulation is meant to represent the actual long term use of the weapon system under real conditions and is essentially producing sample results that could occur if that number of trajectories were actually fired.

2. An approximation to the density function of interest is obtained by constructing a frequency distribution with the numbers of entries in each bin being the number of times that an event (such as the projectile coming to rest at a particular location or the number of times that a person stood at that location would be hit) occurs. The frequency distribution is converted to a histogram that represents probability density by scaling the counts by the number of trajectories and the area of a bin.

3. As an example the method can be applied to illustration 2 from clause 303 as follows:

a. A regular two dimensional grid is constructed over the terrain area of interest to represent histogram bins

$$\left[x_0, x_0 + \Delta x, \ldots, x_0 + n_x \Delta x\right[\otimes \left[z_0, z_0 + \Delta z, \ldots, z_0 + n_z \Delta z\right[\quad (5.1)$$

with the bin ij covering the area

$$\left[x_0 + (i-1)\Delta x, x_0 + i\Delta x\right[\otimes \left[z_0 + (j-1)\Delta z, z_0 + j\Delta z\right[; \quad (5.2)$$

b. The bin counts are initialized to zero;

c. A large number, 1 000 000 say, of trajectories are calculated:

(1) An elevation E and an azimuth A are generated from their probability distributions;

(2) The trajectory is calculated for the elevation and azimuth to obtain the impact location (x, z) on the terrain surface;

(3) The count is incremented by 1 for bin ij that contains this location with

$$i = \frac{x - x_0}{\Delta x} + 1$$
$$j = \frac{z - z_0}{\Delta z} + 1 \quad (5.3)$$

d. The bin counts are scaled by the number of trajectories, 1 000 000, and the area of a bin $\Delta x \Delta z$ so that the approximation represents a probability density function that integrates to 1.

0504. IMPORTANCE SAMPLING

1. Direct sampling, as used in Monte Carlo simulation, is not very efficient where the probabilities of interest are small. For example, even for simple problems, to obtain reliable approximations to a probability of 1 in 1 000 000 it is necessary to carry out a simulation with many more than 1 000 000 trajectories.

2. One method of obtaining more reliable results is to use a modification of the sampling method that is called importance sampling. Instead of generating random numbers from the underlying distributions we generate uniform random numbers over the range of definition of the distribution and apply a weighting factor to the sampled values. When the weighting factor is chosen appropriately the method produces reliable results even for low probabilities. A simulation with 1 000 000 trajectories can provide accurate results for all probabilities down to 1 in 1 000 000.

3. The method still produces an approximation to the density function of interest that is obtained by constructing a frequency distribution with the entries in each bin being the weighting factors accumulated as the event (such as the projectile coming to rest at a particular location) occurs. The frequency distribution is converted to a histogram that represents probability density by scaling the values by the sum of the weighting factors and the area of a bin.

4. The method is applied to illustration 2 from clause 303 as follows:

c. A regular two dimensional grid is constructed over the terrain area of interest to represent histogram bins

$$\left[x_0, x_0 + \Delta x, \ldots, x_0 + n_x \Delta x\right[\otimes \left[z_0, z_0 + \Delta z, \ldots, z_0 + n_z \Delta z\right[\quad (5.4)$$

with the bin ij being covering the area

$$\left[x_0+(i-1)\Delta x, x_0+i\Delta x\right[\otimes \left[z_0+(j-1)\Delta z, z_0+j\Delta z\right[; \qquad (5.5)$$

d. The bin counts are initialized to zero;

e. A large number, 1 000 000 say, of trajectories are simulated:

(1) An elevation E is generated uniformly from the range of elevations $E_{\min} \le E \le E_{\max}$ and an importance weight $w_E = f_E\left(E \middle| \alpha_E, \beta_E, \cdots\right)$ is calculated where $f_E\left(E \middle| \alpha_E, \beta_E, \cdots\right)$ is the probability density function of elevation with parameters $\alpha_E, \beta_E, \cdots$.

(2) An azimuth A is generated uniformly from the range of elevations $A_{\min} \le A \le A_{\max}$ and an importance weight $w_A = f_A\left(A \middle| \alpha_A, \beta_A, \cdots\right)$ is calculated where $f_A\left(A \middle| \alpha_A, \beta_A, \cdots\right)$ is the probability density function of elevation with parameters $\alpha_A, \beta_A, \cdots$.

(3) The trajectory is calculated for the elevation and azimuth to obtain the impact location (x, z) on the terrain surface;

(4) The count is incremented by $w = w_E w_A$ for bin ij that contains this location with

$$i = \frac{x - x_0}{\Delta x} + 1$$
$$j = \frac{z - z_0}{\Delta z} + 1 \qquad (5.6)$$

d. The bin counts are scaled by the sum of the bin counts and the area of a bin $\Delta x \Delta z$ so that the approximation represents a probability density function that integrates to 1.

0505. DATA CUBES

1. While the term “Data Cube” is applied to the various subcomponents of the WDA/Z weapon model, the actual data cube is comprised of both organized multi-dimensional look-up tables that may sample the existing weapon physical performance data and/or coefficient terms describing a polynomial approximation to a physical characteristic specific to the weapon’s unique behaviour.

2. Data Cube development begins with developing statistical models for weapon delivery accuracy, ricochet, and weapon failures modes using both existing data and analytical models. These models consist of the data necessary to determine the initial impact distribution, the ricochet impact distribution, and the failure mode impact distribution. These individual distributions are used to generate a combined distribution that defines the potential impacts of a specific weapon release.

3. The primary impact data cube query yields the standard deviation of the one or more distributions used to describe the weapon initial impact distribution. The initial impact distribution represents the expected nominal weapon behaviour that results in an impact at or about the intended target. If the weapon has a failure mode this failure mode impact distribution is placed along the nominal weapon trajectory and combined with the initial impact distribution. From the expected weapon release location, a secondary calculation is performed using ballistic terms that model the failure behaviour of the weapon. This failure mode ballistic calculation is used to determine the mean impact location for the failure mode impact distributions.

4. In the situation where the modelled impact results in a potential ricochet, the resulting distribution is placed relative to the impact location that produced the ricochet condition. The subsequent ricochet ballistic calculation uses input conditions derived from the calculated terminal impact velocity and angle determines the exit angle and velocity for the specified weapon and impact surface type. The ricochet distribution data cube is queried with the ricochet velocity and angle to determine the mean and standard deviation of the ricochet distribution.

5. Guided Weapons (GW) use datasets similar to those used for unguided weapons but these more complex datasets also must account for failure mechanisms associated with a sophisticated and weapon-unique autonomous guidance and flight control system. The development of GW data cubes starts with a sensitivity analysis to evaluate the GW sensitivity to airspeed, altitude, release angle, target offset, and slant range. Once sensitivity is defined, a series of simulation scenarios can be developed that will provide the data necessary to develop a failure mode data cube. Generally a separate data cube will need to be developed for each failure mode effect. Additional information on GW is provided in ARSP-2 Volume III.

0506. COMBINED CALCULATIONS

1. The use of any of these methods is not practical for some models. As an example the application of importance sampling to all the components of a ricochet model when a trajectory can have several impacts (for example in ricochet off water) would be difficult. The use of multiple samples at each impact would result in a program having to handle too many combinations, for instance 100 samples at each of three impacts would result in 1 000 000 trajectories for each initial impact. Hybrid calculations employ a mixture of the three methods. The efficiency of the hybrid method will depend on the relative use of the different methods.

0507. CALCULATION OF PROBABILITY OF HIT, INJURY OR DEATH

1. The descriptions of the calculation methods above have been applied to the estimation of the distribution of the final resting places of projectiles. The calculation used for the estimation of the probabilities of hit or death that is required as part of the estimation of individual risk is a little more complicated. Whereas the final resting place is usually the direct output from a trajectory calculation the determination of the locations that a person would be hit, given that they were present at those locations, requires the examination of the complete trajectory.

2. A correct calculation would require the intersection of the trajectories with person sized objects to be determined and then a decision would be made as to whether hit would result in injury or death. Given all the uncertainties present in the models used this is not considered necessary and an approximation is made whereby the people are represented by a regular array of cuboids that are positioned on the terrain surface and are aligned with the coordinate axes. Intersections of trajectories with these cuboids are determined by checking for intersections with appropriate planes and the heights above the terrain surface are calculated. Where this height is less than the height of a person a hit is counted and wounding models can then be applied to determine whether an injury or a death should be counted. This approximation using simple geometric shapes simplifies, and hence speeds up, the calculations and produces conservative results in that it overestimates the hazard or risk.

3. The resulting frequencies could be stored as they are calculated but the number of results (one for each cuboid sized bin) would be excessive and an averaging process is generally used. Here the frequencies for a collection of small cells is summed and divided by the number of cells and stored as corresponding to larger cells each of which corresponds to a unique collection of small cells.

4. As an example the method can be applied with Monte Carlo sampling to derive probabilities of hit for a generalization of the problem described in illustration 2 from clause 303 as follows:

a. A regular two dimensional grid is constructed over the terrain area of interest to represent histogram bins

$$\left[x_0, x_0 + \Delta x, \ldots, x_0 + n_x \Delta x\right[\otimes \left[z_0, z_0 + \Delta z, \ldots, z_0 + n_z \Delta z\right[\quad (5.7)$$

with the bin ij covering the area

$$\left[x_0 + (i-1)\Delta x, x_0 + i\Delta x\right[\otimes \left[z_0 + (j-1)\Delta z, z_0 + j\Delta z\right[; \quad (5.8)$$

b. The bin counts are initialized to zero;

c. A large number, 1 000 000 say, of trajectories are simulated:

(1) An elevation E and an azimuth A are generated from their probability distributions;

(2) The trajectory is calculated for the elevation and azimuth to obtain the impact location (x,z) on the terrain surface. The intersections of the trajectory with the a grid of planes constructed over the terrain area of interest to represent an array of people stood on the terrain surface, each one assumed to cover an area of $\Delta p \times \Delta p$;

$$\left[x_0, x_0 + \Delta p, \ldots, x_0 + n_{xp}\Delta p\right[\otimes \left[z_0, z_0 + \Delta p, \ldots, z_0 + n_{zp}\Delta p\right[\quad (5.9)$$

(3) Where the difference between the heights of the intersections $y_i(x,z)$ and the terrain surface at the same (x,z) location $y_T(x,z)$ is less than the height of a person the hit count is incremented by 1 for bin ij that contains this location with

$$i = \frac{x - x_0}{\Delta x} + 1$$
$$j = \frac{z - z_0}{\Delta z} + 1 \quad (5.10)$$

d. The bin counts are scaled by the number of trajectories, 1 000 000 and the number of cuboids in a bin $\Delta x \Delta z / \Delta p^2$ so that the approximation represents a density function of hits. Note that the density function will not generally represent a probability density as each individual trajectory will almost certainly result in multiple hits.

5. To estimate probabilities of death or injury, as opposed to probabilities of hit, a wounding model is used at step (3) and the counts are only incremented where a hit results in a death or injury.

6. The use of importance sampling, rather than Monte Carlo sampling, for these calculations is simple to implement and is not described here.

0508. HISTOGRAMS AND FREQUENCY POLYGONS

1. Whichever calculation and method is used for the simulation the result is a two dimensional histogram representation of a density function defined over the area of interest. This is a piecewise constant representation i.e. the value of the function is assumed constant in each bin.

2. A better representation can be obtained by simply assuming that the value of the function at the centre of each bin is equal to that of the histogram and taking the function to be linear between each of these bin centres. This representation is known as a frequency polygon. It inherits the total probability present in the histogram i.e. if the histogram represents a probability density function and integrates to 1 then so does the frequency polygon.

3. The use of a frequency polygon is a simple method for smoothing the result. More sophisticated methods are described in References 21 and 22.

a. One method that has been found to be suitable with the methods used for the calculation of WDA/Z is the average shifted histogram (ASH). It is a method that can be applied in post-processing and takes as its input the histogram that is produced by the simulation. It is simple and so quick to use that different levels of smoothing can be compared and a suitable level chosen.

b. More sophisticated methods have the drawback that they are applied to the original impacts (say) and this means that all results have to be produced and stored before the methods are applied.

c. One note of caution should be mentioned — smoothing removes discontinuities in the results and in some cases the discontinuities are important. As an example, where a small arms range has a stop butt there may be an area behind the stop butt where no projectiles land and smoothing applied to results for this case could result in this feature of the problem being hidden.

0509. COMBINING HISTOGRAMS OR FREQUENCY POLYGONS

1. Where it is necessary to combine histograms (or frequency polygons) as part of either the calculation of individual risk or the development of a histogram (or frequency polygon) for a composite WDA/Z this can be done by applying the appropriate probability rules to all the values for individual bins.

2. Assuming that we have two histograms representing independent probabilities $P(A)$ and $P(B)$ the following calculations can be carried out:

a. The probability that both A and B occur

$$P(A \text{ and } B) = P(A)\cdot P(B). \tag{5.11}$$

b. The probability that A or B (exclusively) occur

$$P(A \text{ or } B) = P(A) + P(B) - 2\,P(A \text{ and } B). \tag{5.12}$$

c. The probability that A and/or B occur

$$P(A \text{ and/or } B) = P(A) + P(B) - P(A \text{ and } B). \tag{5.13}$$

d. The probability that A or B occur for mutually exclusive events i.e. where A and B cannot occur together

$$P(A \text{ xor } B) = P(A) + P(B). \tag{5.14}$$

e. The lower bound on the probability

$$P\{\alpha A + (1-\alpha)B\} \geq \min\{P(A), P(B)\}. \tag{5.15}$$

f. The upper bound on the probability

$$P\{\alpha A + (1-\alpha)B\} \leq \max\{P(A), P(B)\}. \tag{5.16}$$

3. The domains of definition for the combined probabilities or frequencies are not necessarily the same as that of the original probabilities or frequencies. Assuming that the domains of definition for A and B are those shown in Figure 5.1 the domain that results from the application of a. is shown in Figure 5.2, and the domains that result from the application of b., c., and d., e., and f. are shown in Figure 5.3.

4. Each of the rules can be applied repeatedly. For example:

$$\begin{aligned} P(A \text{ and } B \text{ and } C) &= P\{(A \text{ and } B) \text{ and } C\} \\ &= P(A \text{ and } B) \cdot P(C) \\ &= P(A) \cdot P(B) \cdot P(C) \end{aligned} \quad . \tag{5.17}$$

5. Where the different rules are combined the order that they are applied in is important as the rules are not associative. For example:

$$P\{(A \text{ and } B) \text{ or } C\} \neq P\{A \text{ and } (B \text{ or } C)\}. \tag{5.18}$$

6. Frequencies can be handled by allowing scaling factors to be used with the probabilities A and B. Exposure and the presence of people can be handled by converting areas/zones where people are present into equivalent grids with appropriate values in the grid bins. A combination of these processes and repeated application of the rules can then be used to process histogram (or frequency polygon) representations of the terms that are components of individual risk.

0510. SMEARING HISTOGRAMS OR FREQUENCY POLYGONS

1. The purpose of smearing is to expand the distribution to cover uncertainties in quantities such as launch location, launch heading. Smearing is a step that should only be performed once in the density function generation process. If the desired output is the unprocessed density function, showing the degrees of risk through the interior of the WDA/Z, the smear step must be performed before a WDA/Z is generated. If the desired output is a boundary generated by a particular risk criteria it could be performed following the generation of a WDA/Z instead.

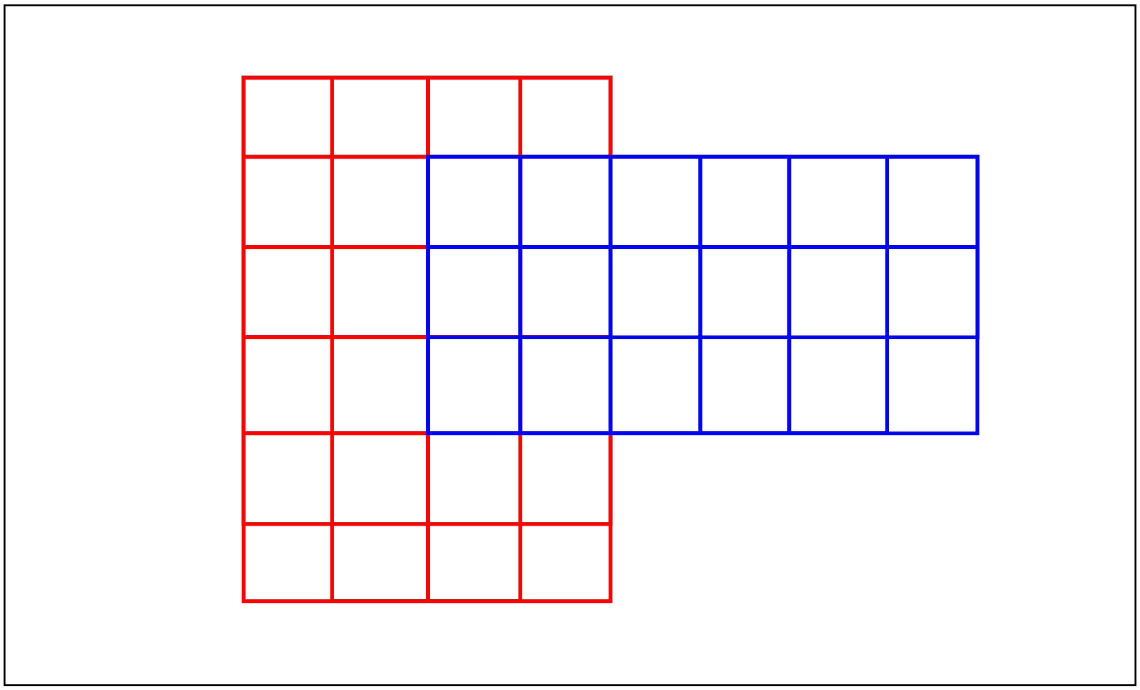

Figure 5.1 — Domains of definition for P(A) (blue) and P(B) (red).

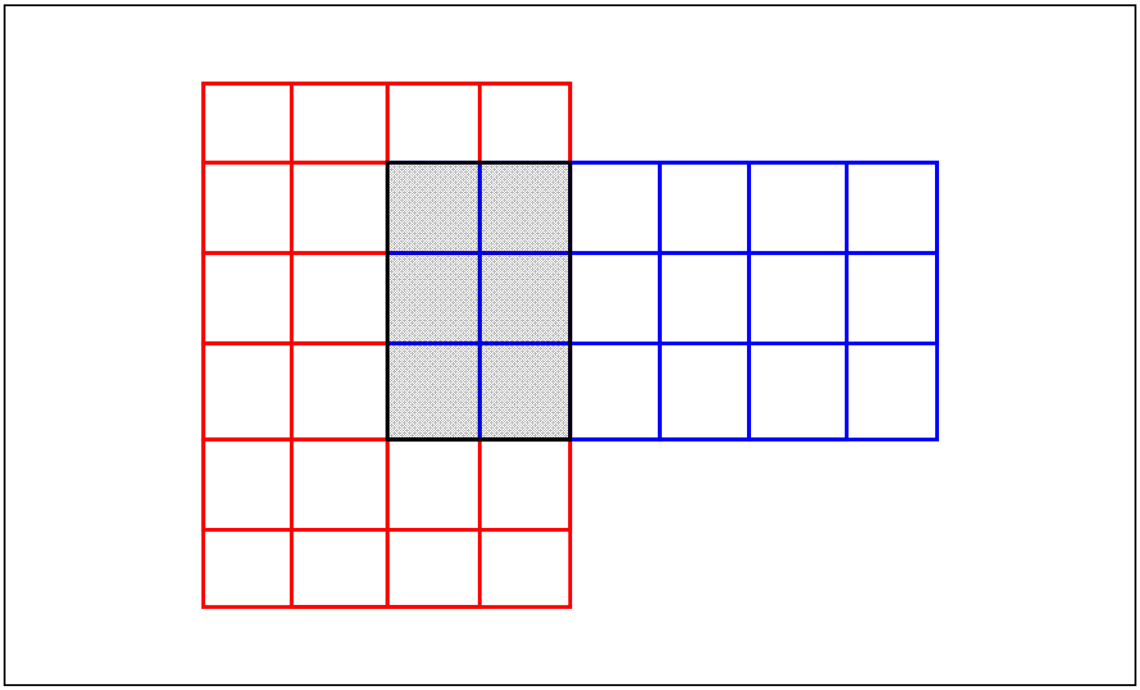

Figure 5.2 — Domain of definition for P(A and B) (black outline/grey shading) over domains of definition for P(A) (blue) and P(B) (red).

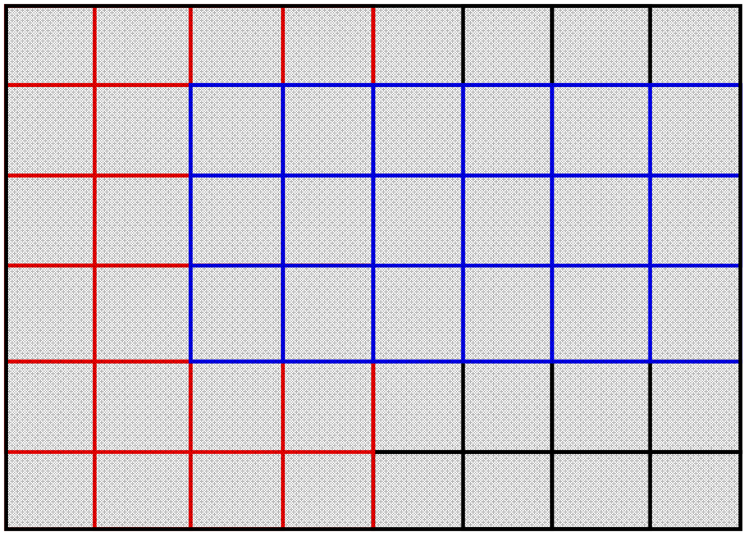

Figure 5.3 — Domain of definition for P(A or B), P(A xor B), min[P(A),P(B)] and max[P(A),P(B)] (black outline/grey shading) over domains of definition for P(A) (blue) and P(B) (red).

CHAPTER 6
CALCULATION OF HAZARD AND RISK FROM DENSITY FUNCTIONS

0601. *INTRODUCTION*

1. This chapter is concerned with the calculation of hazard, or risk, associated with a specified area or volume. Given a frequency polygon approximation to a density function defined on a regular grid and an area or volume we have to calculate values associated with that area or volume.

2. The calculation takes two forms, which may depend on what the density function represents. Firstly, we can calculate the density function values at the boundary of the area or volume. Secondly, we can calculate the probability of being inside or outside the area. The probability of being inside the area is calculated by integrating the density function over the area and the probability of being outside the area is calculated by subtracting this from the total probability. Note that integrals over volumes do not usually have any meaning and are not considered here.

3. A list of options for calculating hazard and risk together with measures of risk (from Chapter 2) is provided in Annex D.

0602. *CALCULATIONS FOR A SIMPLE POLYGON*

1. In principle it is possible to calculate values for any simple area, i.e. any region enclosed by a single closed curve that does not intersect itself. From a practical view it is better to restrict the areas to simplify the calculations that need to be carried out. Here we limit the area to being that represented by a simple polygon i.e. a single closed polygon that does not intersect itself. An example of a simple polygon is shown in Figure 6.1.

2. There are an infinite number of values around the edges of a polygon and hence some summary statistics of the values have to be calculated for presentation. The following statistics can be calculated:

   a. The minimum value;

   b. The maximum value;

   c. The average value;

   d. The standard deviation of the values.

3. In order to calculate the probability of being inside a simple polygon it is necessary to integrate the density function over the polygon. It should be noted that the total probability is obtained by simply summing all the values in the grid. Once the probability of being inside is obtained the probability of being outside is obtained by subtracting this from the total probability.

4. In order to carry out either of the calculations it is necessary to partition the polygon edges into line segments that intersect the edges of the grid cells. The density function is assumed to be bilinear in each grid bin and once the polygon edges have been partitioned each grid bin can be processed on its own. An example of the partition of a single edge of a polygon is shown in Figure 6.2.

5. Values around the edge of the polygon are calculated by processing individual line segments:

   a. The minimum and maximum values around the polygon are the minimum and maximum of the values at the ends of the partitioned line segments;

   b. The average and standard deviation of the values around the polygon can be obtained by integrating along each partitioned line segment and summing these for all line segments.

   c. Integrals over individual grid cells are simple to calculate:

6. For cells with no partitioned line segments in them the integral over the bin is equal to the sum of the four values at the bin vertices multiplied by the area of the bin.

7. Cells with a single partitioned line segment in them are split into two, and this produces either two quadrilaterals or a quadrilateral and a triangle (illustrated in Figure 6.3). As for the complete grid bin the integral over either a quadrilateral or a triangle is equal to the sum of the values at the bin vertices multiplied by the area. The area inside or outside the polygon can be calculated by combining these results in an appropriate way.

8. For cells that have internal vertices, i.e. contain one or more of the vertices of the original polygon, the bin is broken down into cells without internal vertices so that a. and b. apply.

0603. *CALCULATIONS FOR A SIMPLE POLYHEDRON*

1. As with the two dimensional case, it is possible in principle to calculate values for any simple volume, i.e. any region enclosed by a single closed surface that does not intersect itself. It is again more practical to restrict the volumes to simplify the calculations that need to be carried out and now we limit the volume to being that represented by a simple polyhedron i.e. a single closed polyhedron that does not intersect itself.

2. As with simple polygons there are an infinite number of values on the surface edges of a polyhedron and hence some summary statistics of the values have to be calculated for presentation. The following statistics can be calculated:

a. The minimum value;

b. The maximum value;

c. The average value;

d. The standard deviation of the values.

3. Unlike the two dimensional case, there is no point in calculating integrals over the interior or exterior of a polyhedron as the results have no meaning. The three dimensional results have no meaning as, for instance, a projectile that is outside some contour at one location may be inside the contour at another location.

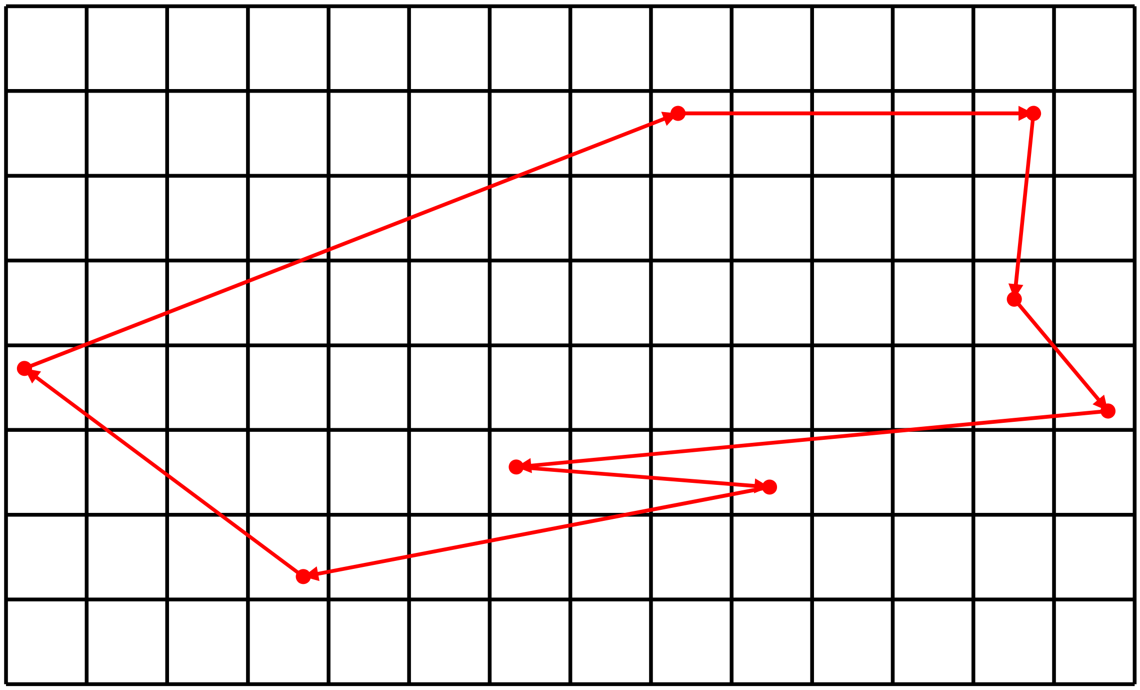

Figure 6.1 — A simple polygon over the grid defining the frequency polygon

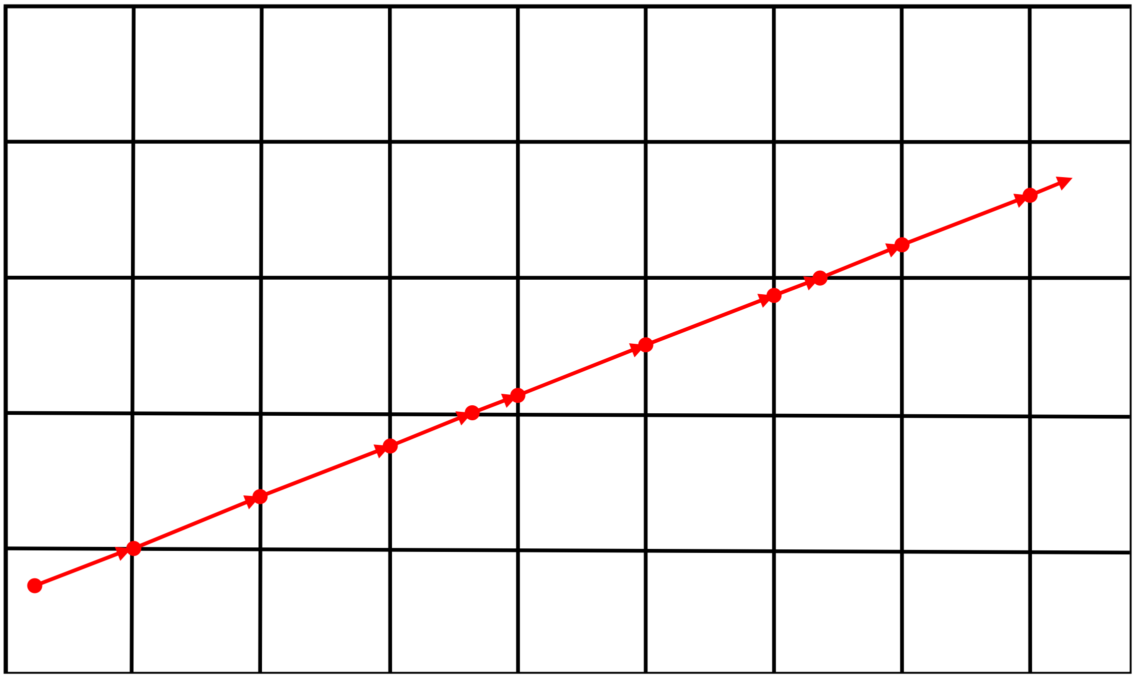

Figure 6.2 — Partition of a single edge of the polygon into line segments

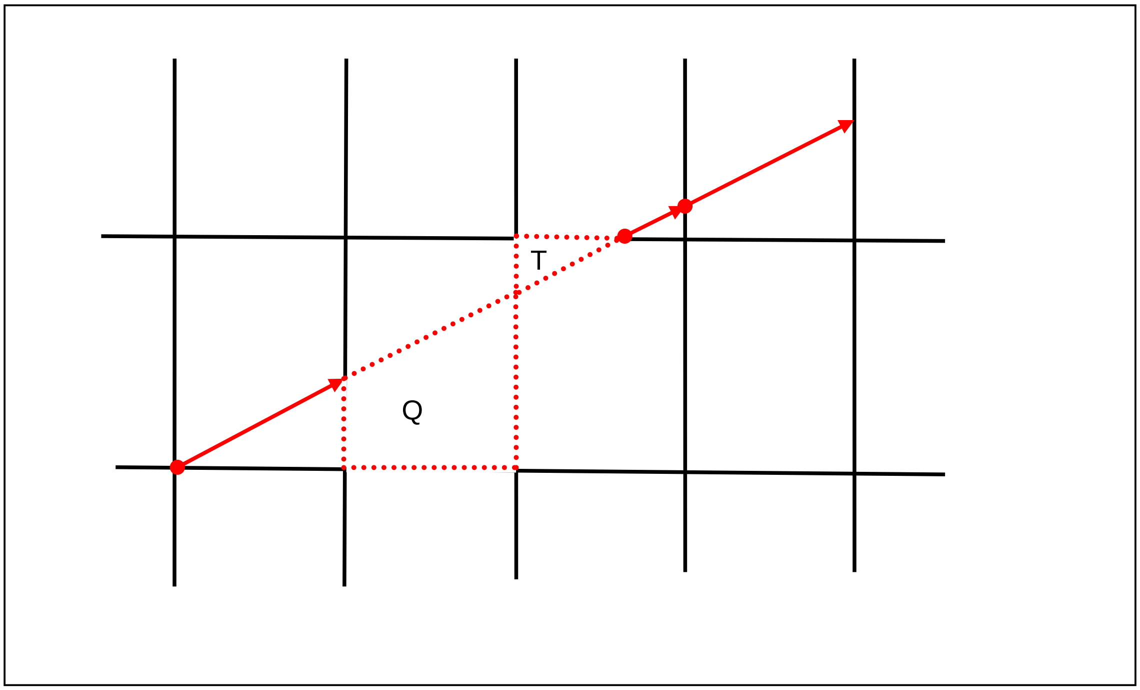

Figure 6.3 — Quadrilateral Q and triangle T used in integration over grid cells.

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CHAPTER 7
DEVELOPMENT OF WDA/Z FROM DENSITY FUNCTIONS

0701. INTRODUCTION

1. This chapter is concerned with the development of WDA/Z from the density functions produced as a result of a simulation or by using data cubes. Given a frequency polygon approximation to a density function defined on a regular grid and some criterion we have to calculate the area or volume corresponding to this criterion.

2. The calculations take two forms. Firstly, we can calculate the area/volume corresponding to a value of the density function. Secondly, we can calculate the area/volume corresponding to the integral of the density function.

3. A list of options for the development of WDA from density functions together with measures of risk (from Chapter 2) is provided in Annex E.

0702. CONTOURS

1. In either case the process revolves around contours of the density function:

   a. The area/volume corresponding to a value of the density function is a contour.

   b. The unique minimum area/volume corresponding to the integral of the density function is also a contour. The contour has to be chosen to obtain the correct area/volumes and the value of the density function has to be determined.

   c. Contours in two dimensions are collections of closed curves, whilst in three dimensions they are collections of closed surfaces.

0703. CONVEX HULLS AND SPECIFIED OUTLINES

1. Because the frequency polygons are produced by simulation they are not usually smooth and hence any contours calculated from them are also not smooth. There are two methods of adapting the contours to produce smoother areas/volumes.

2. Firstly, a convex hull corresponding to the contour can be calculated. This is the boundary of the smallest convex domain containing the contour. With this method it is useful to be able to specify additional points, such as the firing point, that should be included within the convex hull. The convex hull corresponding to a contour in two dimensions and an additional point is illustrated in Figure 7.1.

3. Secondly, a specified shape (such as that corresponding to a WDA/Z developed using deterministic methodology) can be fitted round the contour. With this method the origin of the shape, usually the firing point, have to be specified. This is illustrated in two dimensions in Figure 7.2 where the additional shape corresponds to one in common use for small calibre weapons.

4. When either a convex hull or a specified shape is used it the probabilities associated with the resulting area will be different from that originally specified. Where a value of the density function is specified the use of a convex hull or specified shape will correspond to a lower probability. Where an integral of the density function is specified the use of a convex hull or specified shape will correspond to a higher probability inside and a lower probability outside. In both cases this results in the WDA/Z being larger than necessary.

5. With either a convex hull or a specified shape the probability associated with the shape may be calculated using the methods in clause 602. This means that an iterative procedure can usually be applied to determine the convex hull or specified shape that corresponds to a specified probability. In some cases, for example where the distribution of impacts is not in a single distribution, such a convex hull or specified shape may not exist.

0704. CONTOURS VS. CONVEX HULLS AND SPECIFIED OUTLINES

1. Where a contour is contained by a convex hull or specified outline, as in Figure 7.2, the probability, or frequency, of escape associated with the convex hull or specified outline can be inferred from the contour value. In this case the probability, or frequency, of escape associated with the convex hull or specified outline will always be lower than that for the contour.

2. However, where a contour is not contained by a convex hull or specified outline, as in Figure 7.3, the probability, or frequency, of escape associated with the convex hull or specified outline cannot be inferred from the contour value. There is a common misconception that a particular probability, or frequency, contour crossing a range boundary implies that the probability, or frequency, of escaping from the range must be higher than that for the contour. In reality, the probability, or frequency, of escape associated with the convex hull or specified outline can only be determined by using the methods in Chapter 6.

0705. COMPOSITE WDA

1. Where WDAs are to be developed for composite scenarios the methodology described in clause 308 implemented through the procedures in clause 508 are used to produce a histogram or frequency polygon that is then processed in the same way that an individual histogram or frequency would be processed.

2. Where the WDA is based on probability of escape or individual risk and the frequencies for the individual scenarios is known the individual results are combined with the known frequencies and the combined results are rescaled.

3. Where the WDA is based on probability of escape or individual risk and the frequencies for the individual scenarios are not known the individual results are combined to get the upper bound and the combined results are not scaled.

0706. POPULATION DEPENDENT WDA

1. Where WDAs are to be developed, based on individual risk, and this depends on population the data for the population is converted to a probability density for being present. The procedures in clause 508 are then used to produce a histogram or frequency polygon for the combined result of being hit given present and being present.

2. Where a WDA is used as an exclusion zone, i.e. the population is not meant to be inside it, and any people inside have to move outside, care has to be taken in processing the histograms or frequency polygons. The population probability density has to be continually updated as the processing is carried out to match the real situation.

0707. TWO DIMENSIONAL SUBSETS OF THREE DIMENSIONAL RESULTS

1. It is useful to be able to examine data on planes through three dimensional results. As an example, the results on a vertical plane through a target line on a small arms range can be used to determine the size of a stop butt to match some prescribed probability level. Once a two dimensional subset of three dimensional results has been extracted it can be processed in the same way as any two dimensional result that is directly produced.

0708. WDZ OR ADH FOR ZERO RISK

1. Where a WDZ or ADH is required to match zero risk a simplified calculation can be carried out that avoids post processing a full three dimensional result. As the trajectory calculations are being carried out the maximum height in a grid bin is determined and this is stored rather than a probability. For a WDZ the surface defined by these heights is used, whereas the maximum height over the grid is used for an ADH.

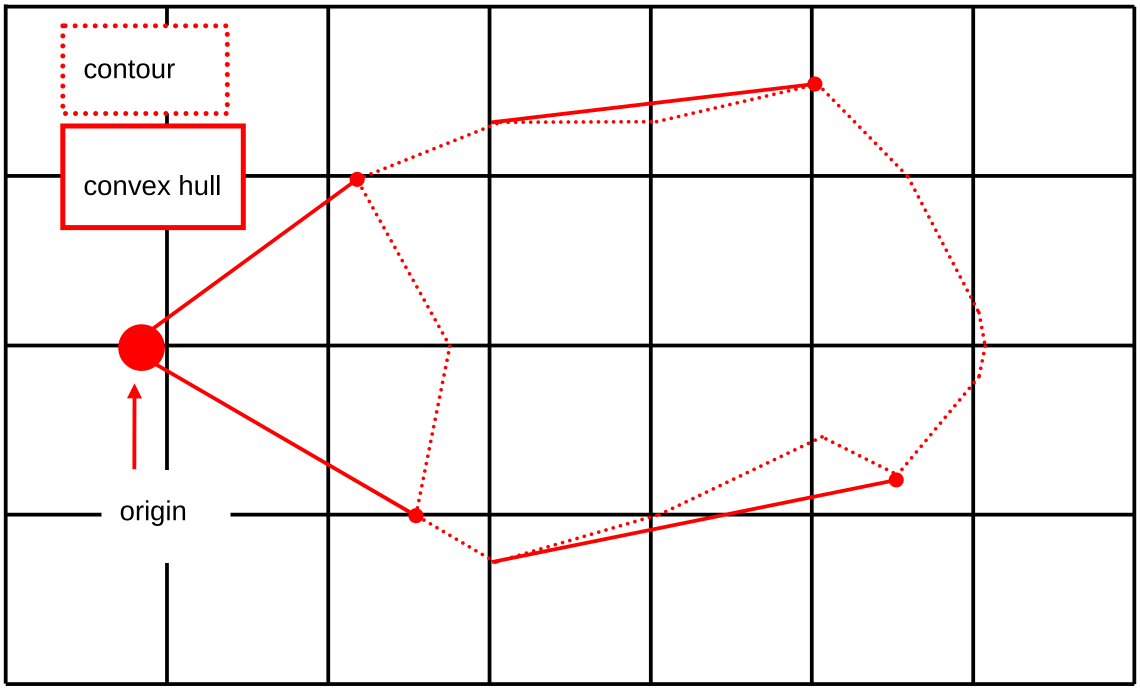

Figure 7.1 — A convex hull corresponding to a contour in two dimensions

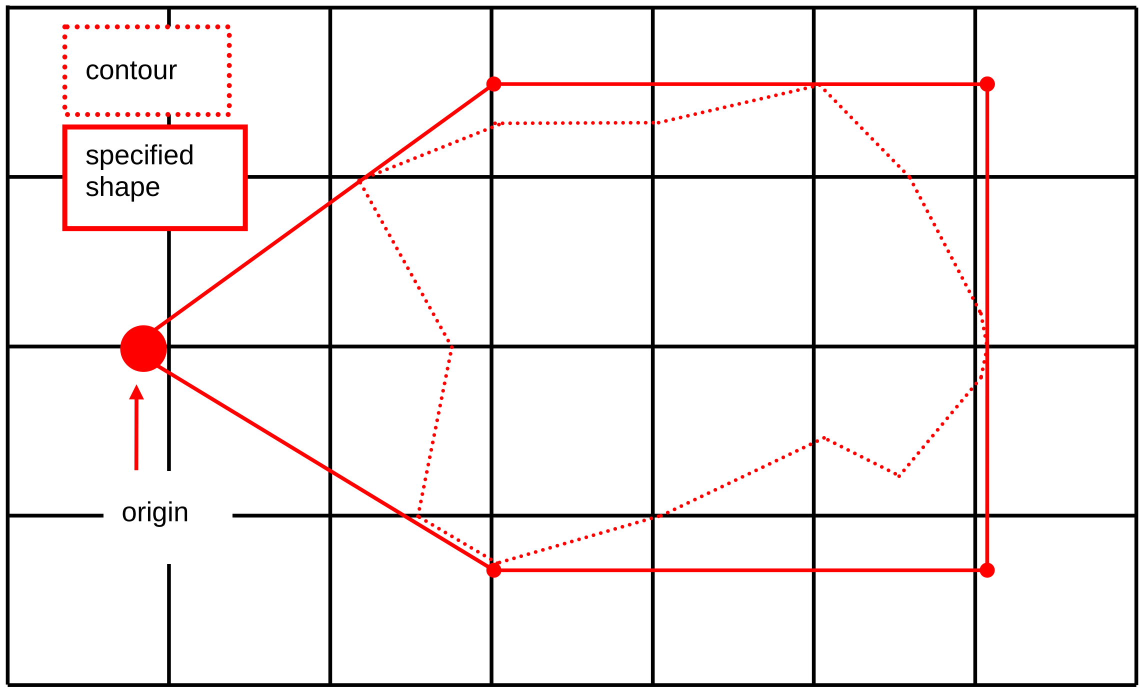

Figure 7.2 — A specified shape covering a contour in two dimensions i.e. the contour is fully enclosed within the shape.

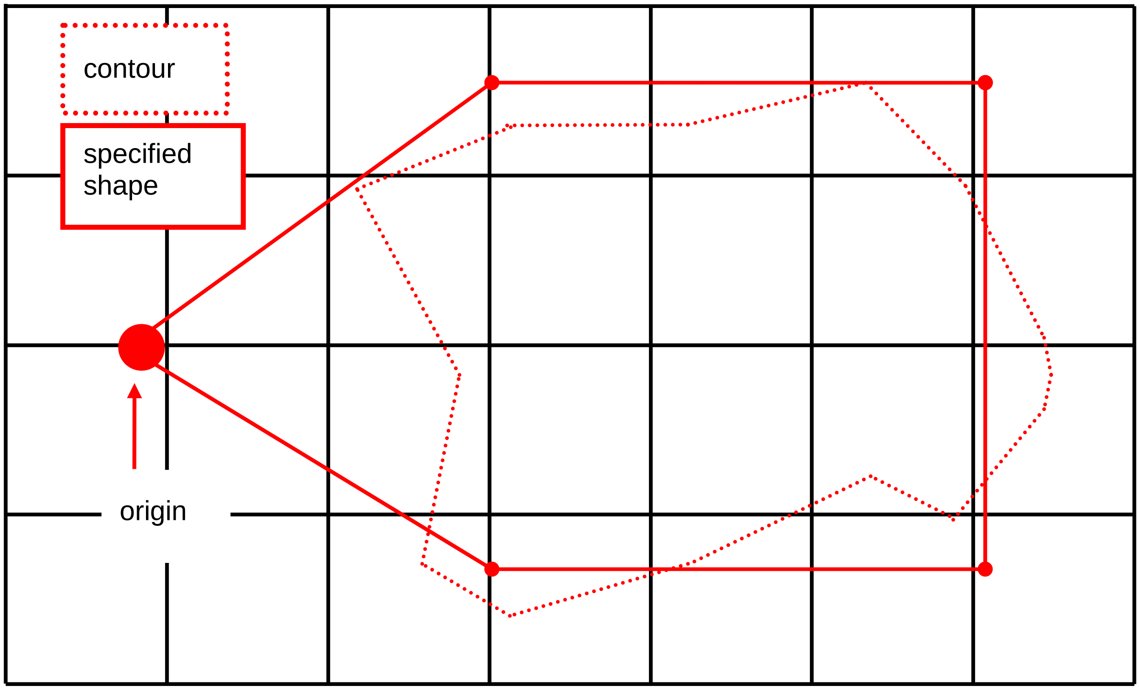

Figure 7.3 — A specified shape partially covering a contour in two dimensions i.e. the contour crosses the boundary of the shape and is not fully enclosed within the shape.

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ANNEX A
PROBABILITY CALCULATIONS — ANALYTIC METHODS

A01. INTRODUCTION

1. In addition to the simulation methods described in Chapter 5 there are analytic methods for deriving the density functions. Here we describe three of these in terms of problems involving point-mass equations

a. Transformation of variables;

b. Liouville's equation;

c. Fokker-Planck equations.

2. The methods are described here for two reasons. Firstly, they demonstrate that formal mathematical derivations can be produced for the problems that we are dealing with but that they do not yield any results that are practical for real problems. Secondly, they produce analytic solutions that can be used in verifying computer implementations of simulation.

3. The transformation of variables method (References 23 and 24) is widely used in mathematical statistics, for instance it is used to derive the chi-squared distribution as the sum of a number of squares of normally distributed variables. It is feasible only for relatively simple functions of random variables but its application to a simple point-mass problem is presented in Annex B.

4. For both Liouville and Fokker-Planck equations solutions are obtained by solving partial differential equations subject to suitable initial and boundary conditions. This is a not inconsiderable task and we have succeeded in replacing one difficult problem with an even more difficult one. However it can be demonstrated that the solution of Liouville's equation for the simple problem involving a point-mass model agrees with that obtained by using transformation of variables.

A02. TRANSFORMATION OF VARIABLES — EXAMPLE 1

1. Initially we look at the problem of a transformation of one variable E to one variable R. The distribution of the input and output variables are specified by their probability density functions:

$$\begin{aligned} E &\sim f_E\left(E \middle| \alpha_E, \beta_E, \cdots\right) \\ R &\sim f_R\left(R \middle| \alpha_R, \beta_R, \cdots\right) \end{aligned} \qquad \text{(A.1)}$$

a. Where $\alpha_E, \beta_E, \cdots$ and $\alpha_R, \beta_R, \cdots$ are the parameters of the distributions.

b. If the output (range) is a given function of the input (elevation) i.e. $R = R(E)$ the two probability density functions are related by:

$$f_R\left(R \middle| \alpha_R, \beta_R, \cdots\right) = f_E\left\{E(R) \middle| \alpha_E, \beta_E, \cdots\right\}\left|\frac{dE}{dR}\right| \qquad \text{(A.2)}$$

2. For this equation to be valid and useful there are three conditions that apply:

a. The function $R(E)$ has to be one-to-one i.e. a unique elevation E has to map to a unique range R;

b. The function $R(E)$ has to be onto i.e. for every R there has to be a corresponding E;

c. $\left|dE/dR\right|$ has to be non-zero for all values of R — in fact either $dE/dR < 0$ or $dE/dR > 0$.

3. Satisfaction of the first and second conditions ensures that the inverse function $E(R)$ exists. The third ensures that the output probability density function is finite. It is easy to construct situations where one or both of these conditions do not hold. The first condition does not hold when we have both low and high angle fire, and two different elevations result in the same range. It is clear that the third condition is equivalent in this case as $|dE/dR|$ is zero at maximum range as we move from low angle fire to high angle fire. The second condition does not hold if we specify a range that cannot be achieved.

4. Assuming that the conditions are met, how do we use the relationship to obtain something that we can use? If the inverse function $E(R)$ and its derivative dE/dR are analytic then the result given by (A.2) will be analytic and can be used. If they are not analytic then we have to resort to numerical methods to construct an approximation to the result.

5. When the conditions are not satisfied the method has to be adapted to overcome this. For this simple problem it is not too difficult to do this. For example where we have both low and high angle the problem can be split into two to obtain density functions for each with the result for the complete problem being obtained by adding the two together.

A03. TRANSFORMATION OF VARIABLES — EXAMPLE 2

1. We now extend the problem by including azimuth and map (E,A) to (x,z). We assume that E and A are independent, the probability density functions for both inputs and the joint probability density function for the outputs are:

$$\begin{aligned} &E \sim f_E\left(E \mid \alpha_E, \beta_E, \cdots\right) \\ &A \sim f_A\left(A \mid \alpha_A, \beta_A, \cdots\right) \\ &(x,z) \sim f_{xz}\left(x,z \mid \alpha_{xz}, \beta_{xz}, \cdots\right) \end{aligned} \tag{A.3}$$

2. If the outputs are given functions of the inputs i.e. $x = x(E,A)$ and $z = z(E,A)$ the probability density functions are now related by:

$$f_{xz}\left(x,z \mid \alpha_{xz}, \beta_{xz}, \cdots\right) = f_E\left\{E(x,z) \mid \alpha_E, \beta_E, \cdots\right\} f_A\left\{A(x,z) \mid \alpha_E, \beta_E, \cdots\right\} |J| \tag{A.4}$$

where J, the Jacobian of the transformation, is the determinant

$$J = \begin{vmatrix} \dfrac{\partial E}{\partial x} & \dfrac{\partial E}{\partial z} \\ \dfrac{\partial A}{\partial x} & \dfrac{\partial A}{\partial z} \end{vmatrix} = 1/J^{-1} = 1/\begin{vmatrix} \dfrac{\partial x}{\partial E} & \dfrac{\partial x}{\partial A} \\ \dfrac{\partial z}{\partial E} & \dfrac{\partial z}{\partial A} \end{vmatrix} \tag{A.5}$$

3. This relationship depends on a number of conditions as before, and as before it is not difficult to construct situations where these conditions do not hold.

4. Even if the result is correct it does not appear at first glance to be of much use. However, for problems where the range $R = \sqrt{x^2 + z^2}$ is independent of azimuth and the elevation as a function of range and its derivative is analytic, use of the intermediate variables R and A allows results to be derived. This is equivalent to working in polar coordinates and the details will not be given here (full details are given in Annex B for a specific example).

5. As with Example 1 the method can be adapted to overcome problems that occur when the conditions are not met but it is now quite easy to specify examples where the method cannot be adapted easily.

6. The method can be used where there are more input variables, such as the launch velocity, but it becomes much more complicated. An auxiliary variable has to be introduced for each extra input and the joint distribution for all variables derived. The auxiliary variables are then eliminated by integration to obtain the required result.

A04. LIOUVILE'S EQUATION AND A FOKKER-PLANCK EQUATION FOR A POINT-MASS MODEL

1. Where the random variation is in the initial values of position and/or velocity a partial differential equation of the Liouville type can be derived. Where there is also random variation in, for example, the aerodynamic forces a Fokker-Planck equation can be derived. The detailed derivation of these equations is beyond this publication and details are in Reference 25. Here a brief summary of the two formulations is given.

2. Consider the motion of a point mass in the 6-dimensional phase space consisting of the Cartesian co-ordinates $\boldsymbol{x} = (x_1, x_2, x_3)$ of position and the corresponding velocity components $\boldsymbol{u} = (u_1, u_2, u_3)$. Let $f(\boldsymbol{x}, \boldsymbol{u}, t)\, d\boldsymbol{x}\, d\boldsymbol{u}$ be the probability that the point mass will be in a small volume of phase space $d\mathbf{x}d\mathbf{u}$ centred on the point $(\boldsymbol{x}, \boldsymbol{u})$ at time t.

3. The actual trajectory of the point mass is represented in this phase space by a curve which is given by its position and velocity vectors $\boldsymbol{X}(t)$ and $\boldsymbol{U}(t)$. We assume that the latter satisfy equations of motion of the form

$$\begin{aligned} \dot{\boldsymbol{X}}(t) &= \boldsymbol{U}(t) \\ \dot{\boldsymbol{U}}(t) &= \boldsymbol{a}\left\{\boldsymbol{X}(t), \boldsymbol{U}(t)\right\} \end{aligned} \qquad \text{(A.6)}$$

a. Where the acceleration $\boldsymbol{a}$ is a given function of position and velocity only.

b. Where the variability arises from launch errors, i.e. variability in the initial values, $\boldsymbol{X}(0)$ and/or $\boldsymbol{U}(0)$ we obtain a Liouville equation

$$\frac{\partial f}{\partial t} + \boldsymbol{u} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \boldsymbol{a} \cdot \frac{\partial f}{\partial \boldsymbol{u}} = Jf \qquad \text{(A.7)}$$

where

$$J(\boldsymbol{x}, \boldsymbol{u}) = \frac{\partial a_i}{\partial u_i} \qquad \text{(A.8)}$$

4. In order to integrate (A.7), we require an initial condition of the form

$$f(\boldsymbol{x}, \boldsymbol{u}, 0) = \mathcal{F}(\boldsymbol{x}, \boldsymbol{u}) \qquad \text{(A.9)}$$

where $\mathcal{F}(\boldsymbol{x}, \boldsymbol{u})$ is a given function, which is to be chosen to describe the probability distribution of launch errors of position and velocity, so it should also satisfy the normalization condition

$$\int \mathcal{F}(\boldsymbol{x}, \boldsymbol{u})\, d\boldsymbol{x}\, d\boldsymbol{u} = 1 \qquad \text{(A.10)}$$

5. If the motion of the point mass is also subject to variability regarding the value of some parameter affecting the acceleration, due to mechanical or aerodynamic imperfections, for instance, then minor modifications to the foregoing are required. Details are given in Reference 25.

6. Where there is also random variation in, for example, the aerodynamic forces the derivation of the Liouville equation fails when the acceleration function $\boldsymbol{a}$ involves variability due to the presence of a random function of time, such as would arise from a turbulent wind affecting the aerodynamic loading. In this case, the evolution equation for $f(\boldsymbol{x},\boldsymbol{u},t)$ is harder to derive and is of the Fokker-Planck type.

7. We consider the simplest situation in which the acceleration consists of the sum of a term of the type just considered, plus a wholly random function of time, $\boldsymbol{b}(t)$, which does not depend on the trajectory variables $\boldsymbol{X}(t)$ and $\boldsymbol{U}(t)$. Then (A.6) becomes

$$\begin{aligned}\dot{\boldsymbol{X}}(t) &= \boldsymbol{U}(t)\\ \dot{\boldsymbol{U}}(t) &= \boldsymbol{a}\left\{\boldsymbol{X}(t),\boldsymbol{U}(t)\right\} + \boldsymbol{b}(t)\end{aligned} \tag{A.11}$$

8. Since $\boldsymbol{b}(t)$ is now a source of variability, its probability distribution $\mathcal{P}\left\{\boldsymbol{b}(t)\right\}$ has to be specified, just as the probability distribution $\mathcal{F}(\boldsymbol{x},\boldsymbol{u})$ of the launch conditions had to be specified in the initial condition (A.9). For the case where $\mathcal{P}\left\{\boldsymbol{b}(t)\right\}$ describes a Gaussian white noise process we obtain the Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \boldsymbol{u}\cdot\frac{\partial f}{\partial \boldsymbol{x}} + \frac{\partial}{\partial \boldsymbol{u}}\cdot\boldsymbol{a}f = \frac{B}{2}\nabla_{\boldsymbol{u}}^2 f, \tag{A.12}$$

where B is a constant. Again, an initial condition is required to solve this equation together with boundary conditions as appropriate.

ANNEX B
APPLICATION OF TRANSFORMATION OF VARIABLES TO A SIMPLE POINT MASS PROBLEM

B01. INTRODUCTION

1. By considering simple versions of trajectory models it is possible to specify problems that are useful for testing computational strategies. They are useful for two reasons. Firstly the problems have analytic, or easily validated, solutions and secondly they do not require the same computational effort as the real problem.

B02. TRAJECTORY EQUATIONS

2. The models are simplified as follows (Reference 26):

   a. Terrain topography — the surface is flat and the initial launch point is at the origin;

   b. Aimer deviation — the initial launch angles are either normally distributed or chosen to make the range to impact normally distributed;

   c. Free flight and meteorology — we use a point-mass model with constant drag coefficient, the vertical component of drag is ignored, and the launch velocity is constant;

   d. Impact and post-impact flight — when the projectile strikes the ground it is assumed to come to rest and no further trajectories occur.

3. The differential equation for free flight is

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -K\dot{x}^2 \\ -g \end{bmatrix} \tag{B.1}$$

with initial conditions for a trajectory being

$$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \dot{x}(0) \\ \dot{y}(0) \end{bmatrix} = V \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \tag{B.2}$$

B03. TRAJECTORY SOLUTION

1. The solution to this initial value problem is

$$\begin{aligned} \dot{x} &= V\cos\theta \exp(-Kx) \\ \dot{y} &= V\sin\theta - gt \\ x &= \frac{1}{K}\ln(1 + KVt\cos\theta) \\ y &= Vt\sin\theta - \frac{1}{2}gt^2. \end{aligned} \tag{B.3}$$

2. The impact position on a flat range is

$$R(\theta) = x(y=0) = \frac{1}{K}\ln\left(1+\frac{KV^2}{g}\sin 2\theta\right). \qquad \text{(B.4)}$$

and the initial elevation required to achieve a particular range is

$$\theta(R) = \frac{1}{K}\arcsin\left\{\frac{g}{KV^2}\left(e^{KR}-1\right)\right\}. \qquad \text{(B.5)}$$

B04. TRANSFORMATION OF VARIABLES

1. The probability density functions for the launch elevation and the range to impact are related by

$$\begin{aligned} &\theta \sim f_\theta\left(\theta \mid \alpha\right) \\ &\theta = \theta(R) \\ &R \sim \left|\frac{d\theta}{dR}\right| f_\theta\left\{\theta(R) \mid \alpha\right\}. \end{aligned} \qquad \text{(B.6)}$$

and

$$\begin{aligned} &R \sim f_R\left(R \mid \varepsilon\right) \\ &R = R(\theta) \\ &\theta \sim \left|\frac{dR}{d\theta}\right| f_R\left\{R(\theta) \mid \varepsilon\right\}. \end{aligned} \qquad \text{(B.7)}$$

2. If we take the range to impact to be normally distributed, i.e. $R \sim N\left(\mu_R, \sigma_R^2\right)$, then the probability density functions for launch elevation and range to impact, written in terms of R, μ_R & σ_R from (B.7), are

$$\begin{aligned} &f_\theta\left(\theta \mid \mu_R, \sigma_R^2\right) = \left|\frac{2V^2\cos 2\theta}{g+KV^2\sin 2\theta}\right| \exp\left[\left\{\ln\left(1+\frac{KV^2}{g}\sin 2\theta\right) - K\mu_R\right\}^2 \Big/ 2K^2\sigma_R^2\right] \\ &f_R\left(R \mid \mu_R, \sigma_R^2\right) = \left(\frac{1}{2\pi\sigma_R^2}\right)^{1/2} \exp\left\{-\frac{\left(x-\mu_R\right)^2}{2\sigma_R^2}\right\}. \end{aligned} \qquad \text{(B.8)}$$

3. If we take the launch elevation to be normally distributed, i.e. $\theta \sim N\left(\mu_\theta, \sigma_\theta^2\right)$, then the probability density functions for launch elevation and range to impact, written in terms of θ, μ_θ & σ_θ from (B.6), are

$$\begin{aligned} &f_\theta\left(\theta \mid \mu_\theta, \sigma_\theta^2\right) = \left(\frac{1}{2\pi\sigma_\theta^2}\right)^{1/2} \exp\left\{-\frac{\left(\theta-\mu_\theta\right)^2}{2\sigma_\theta^2}\right\} \\ &f_R\left(R \mid \mu_\theta, \sigma_\theta^2\right) = \left|\frac{g+KV^2\sin 2\theta}{2V^2\cos 2\theta}\right| \left(\frac{1}{2\pi\sigma_\theta^2}\right)^{1/2} \exp\left\{-\frac{\left(\theta-\mu_\theta\right)^2}{2\sigma_\theta^2}\right\}. \end{aligned} \qquad \text{(B.9)}$$

B05. VARIABILITY — PROBABILITY CONTOURS

1. The problem where the range is normally distributed is a useful test case for variability because the fractiles of the output are easily obtained as contours. We use the problem for normally distributed range combined with a launch azimuth that is normally distributed. The fractiles of the output distribution are ellipses in range and azimuth:

$$\left(\frac{R-\mu_R}{\sigma_R}\right)^2+\left(\frac{\phi-\mu_\phi}{\sigma_\phi}\right)^2=U_{\chi^2}\left(q\middle|2\right) \qquad \text{(B.10)}$$

where $U_{\chi^2}\left(q\middle|2\right)$ is the upper fractile of the chi-squared distribution, with two degrees of freedom, corresponding to distribution function complement q.

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ANNEX C
APPLICATION OF SIMULATION TO A SIMPLE POINT-MASS PROBLEM

C01. INTRODUCTION

1. We illustrate that the use of simulation provides reasonable approximations to the true solutions as derived by the analytic methods. The results are for the problem defined in Annex B.

C02. VARIABILITY

1. Importance sampling with 1 000 000 trajectories with the launch elevation distribution from (B.8) and the normal distribution for the azimuth was used to derive a numerical approximation to the output distribution. The parameter values used were $\mu_R = 1000$ m, $\sigma_R = 100$ m, $\mu_\phi = 0$ mils, and $\sigma_\phi = 100$ mils.

2. Figure C.1 shows comparison between the 1 in 100, 1 in 10 000, and 1 in 1 000 000 contours obtained using the simulation and the corresponding contours from (B.10). As can be seen the contours obtained by weighted simulation are in good agreement with the correct results. Thus the use of weighted simulation provides an effective method of calculating the required results.

3. Similar results can be obtained using Monte Carlo sampling but much larger sample sizes are required to obtain the same accuracy — for example we would need to use sample sizes approximately 1 000 times as large to get the same agreement for the 1 in 1 000 000 contour.

C03. UNCERTAINTY

1. The problem where the launch elevation is normally distributed is a useful test case for uncertainty because the distributions of the sample statistics are known. The example is based upon the problem for normally distributed launch elevation combined with a launch azimuth that is normally distributed.

2. The population parameters are set to $\mu_\theta = 20$ mils, $\sigma_\theta = 2.90$ mils and $\mu_\theta = 20$ mils, $\sigma_\phi = 100$ mils. Importance sampling using 107 trajectories is used to obtain reference results for these parameters. One hundred samples of size 40 were generated from the underlying distributions, and of these one with large variability (i.e. the sample statistics were a poor representation of the true population parameters) was chosen for use in illustrating the method of dealing with uncertainty. The sample statistics were $\bar{x}_\theta = 19.3$ mils, $s_\theta = 2.49$ mils, $\bar{x}_\phi = 37.38$ mils, and $s_\phi = 98.96$ mils.

3. Calculation of a variability problem using these values merely produces the correct result for a problem where the population parameters are equal to the sample statistics. A set of results are shown in Figure C.2 and as can be seen the solutions obtained are a poor estimate of the true population results.

4. In sampling from the original normal distributions the sample mean and variance have distributions:

$$\bar{x} \sim N\left(\mu, \sigma^2 / n\right)$$
$$(n-1)\frac{s^2}{\sigma^2} \sim \chi^2(n-1). \qquad \text{(C.1)}$$

5. The method used to construct the confidence limits on the fractiles is based upon an attempt to reverse the original sampling process. We manipulate (C.1) to obtain

$$\mu \sim N\left(\overline{x}, s^2 / n\right)$$
$$\sigma^2 \sim \frac{(n-1)s^2}{\chi^2(n-1)} \tag{C.2}$$

and generate samples from these distributions for use in individual variability calculations.

6. We use importance sampling and calculate K = 1, M sets of parameters. For the population means for elevation

$$u_K \sim U\left\{L_N\left(\varepsilon / 4 \middle| 0,1\right), U_N\left(\varepsilon / 4 \middle| 0,1\right)\right\}$$
$$\mu_{\theta K} = \overline{x}_\theta + u_K \cdot \frac{s_\theta}{\sqrt{n}} \tag{C.3}$$
$$w_K = \exp\left(-u_K^2 / 2\right)$$

where $L_N(\varepsilon/4|0,1)$ and $U_N(\varepsilon/4|0,1)$ are the practical limits of the standard normal distribution, which are calculated as the lower and upper fractiles corresponding to one quarter of the machine precision ε .

7. For the population variances for elevation

$$u_K \sim U\left\{L_{\chi^2}\left(\varepsilon / 4 \middle| n-1\right), U_{\chi^2}\left(\varepsilon / 4 \middle| n-1\right)\right\}$$
$$\sigma_{\theta K}^2 = \frac{(n-1)s_\theta^2}{u_K} \tag{C.4}$$
$$w_K = u_K^{\left(\frac{n-1}{2}-1\right)} \cdot \exp\left(-u_K / 2\right)$$

where $L_{\chi^2}(\varepsilon/4|n-1)$ and $U_{\chi^2}(\varepsilon/4|n-1)$ are the practical limits of the chi-squared distribution with n-1 degrees of freedom, which are calculated as the lower and upper fractiles corresponding to one quarter of the machine precision.

8. The same sampling processes are used for the launch azimuths and the weight for each set of parameters is the product of all four individual weights. A variability calculation is carried out for each sample set to obtain a density estimate and we obtain M density estimates from which we can calculate M estimates for the fractiles of interest.

9. The first 25 contours obtained from M = 1 000 variability calculations for the 1 in 1 000 000 contour are shown in Figure C.3. Trying to make sense of such plots is difficult and in fact we only ever use them to illustrate the process used to build the confidence distribution of the contours. Marked on the plot are two regions, one in the interior of the contours shown (marked I) and one that is outside of the contours shown (marked E). Points on the outside are outside all the contours and points in the interior region are inside most of them — note that there may not be any points that are inside all of them! An empirical distribution function is constructed by calculating, at each point on a grid, the fraction of contours that points are interior to:

a. Initialize the empirical distribution function $C_{ij} = 0$

b. For each contour K:

(1) Find the contour value $\left(f_\alpha\right)_K$ corresponding to the probability of interest

(2) For all points in the grid increment the empirical distribution function if the point is inside the contour if $f_{ij} \geq \left(f_\alpha\right)_K$ then $C_{ij} = C_{ij} + w_K$

c. Scale the empirical distribution function $0 \leq C_{ij} = \dfrac{C_{ij}}{\sum w_K} \leq 1$

10. The empirical distribution function values lie between 0 in the exterior, and ~1 in the interior and contours in this function are used as confidence limits. For example, if we want the 95% confidence limit we calculate the contour $C_{ij} = 0.05$.

11. A set of confidence limits, for confidences 50%, 75%, 90%, 95%, and 99% for the 1 in 1 000 000 contour are compared with the true population contour in Figure C.4. The confidence limits straddle the true contour with both the 95% and 99% confidence limits being outside it.

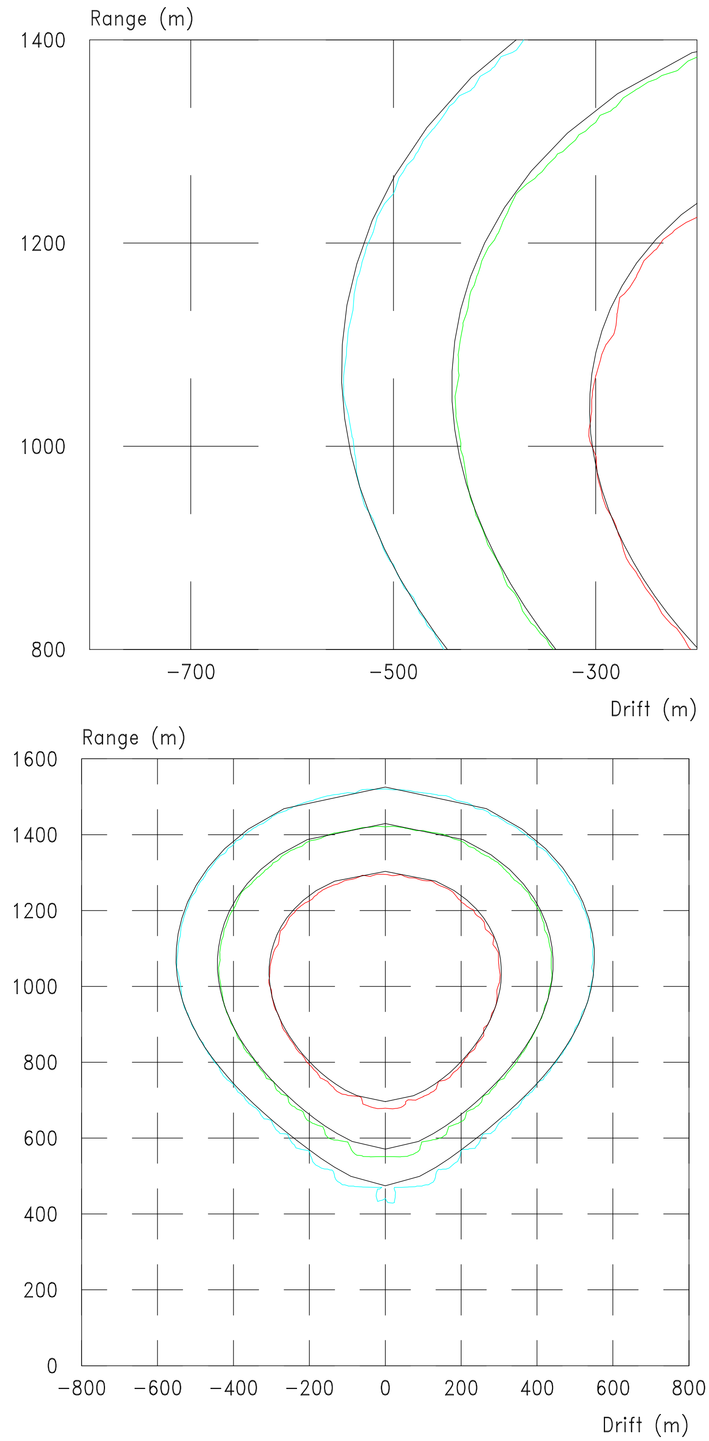

Figure C.1 — Variability problem — Comparison of fractiles obtained by weighted simulation (red 1 in 100, green 1 in 10 000, blue 1 in 1 000 000) with analytic solutions (all in black).

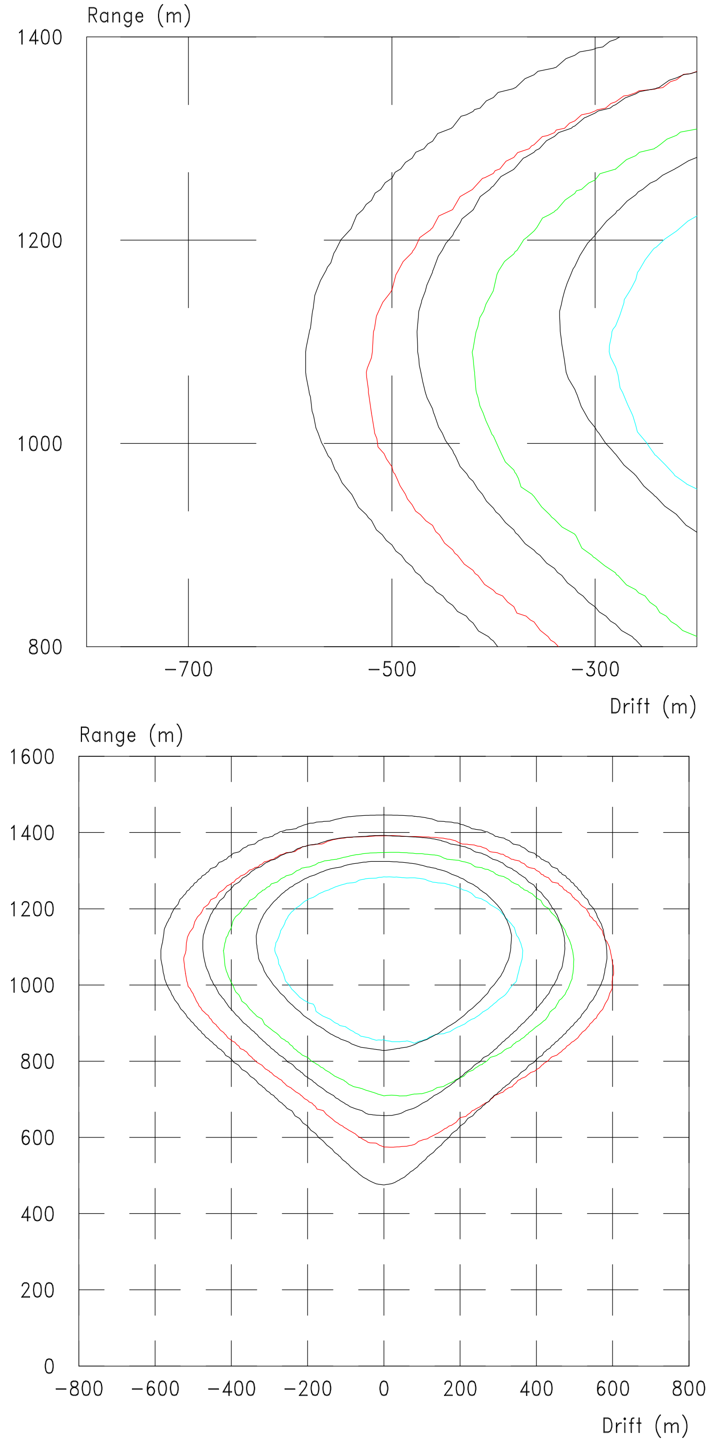

Figure C.2 — uncertainty problem — comparison of fractiles obtained by weighted simulation using sample statistics (red 1 in 100, green 1 in 10 000, blue 1 in 1 000 000) with reference solutions (all in black).

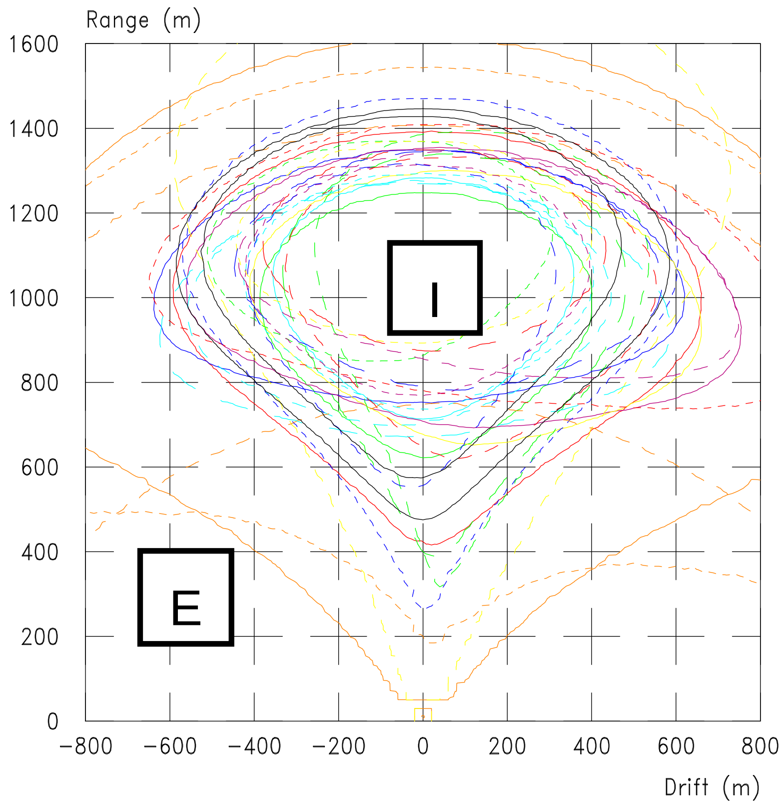

Figure C.3 — Uncertainty problem — the first 25 of a sample of M = 1000 simulations 1 in 1 000 000 contours (colour) compared with the population and sample 1 in 1 000 000 contours (black).

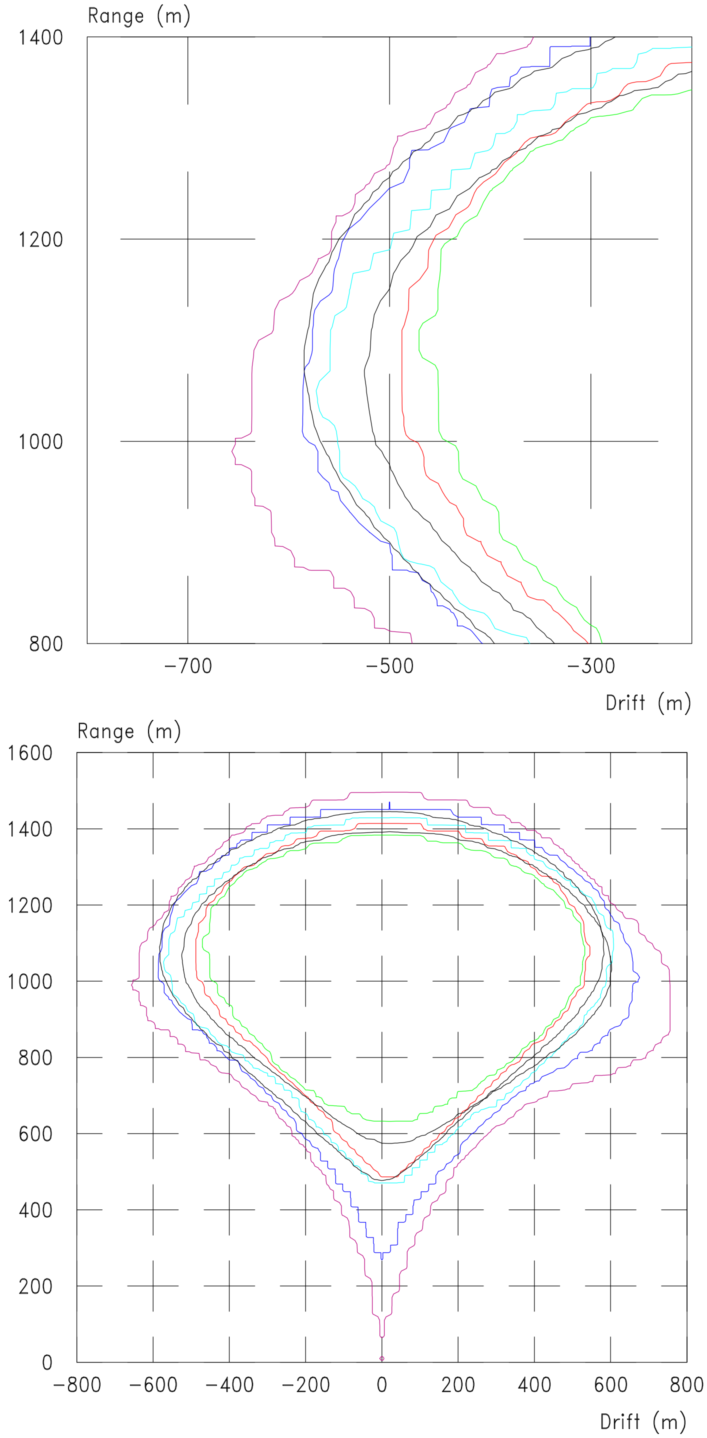

Figure C.4 — uncertainty problem — comparison of confidence limits obtained by weighted simulation (green 50%, red 75%, turquoise 90%, blue 95% and purple 99%) with the population and sample1 in 1 000 000 contours (both in black).

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ANNEX D
OPTIONS FOR CALCULATION OF HAZARD AND RISK FROM DENSITY FUNCTIONS

D01. INTRODUCTION

1. This annex provides details of the options for the calculations of hazard and risk in Chapter 6 using measures of hazard and risk in Chapter 2. For each of the combinations we list the data to be provided and the results obtained.

2. The measures of hazard and risk using impact density are:

   a. Impact density – ID – in impacts per square metre;

   b. Probability of escape – P(E|R) – in escapes per round;

   c. Frequency of escape – F(E|R) – in escapes per round;

   d. Event frequency of escape – F(E|E) – in escapes per event;

   e. Annual frequency of escape – F(E|Y) – in escapes per year.

3. Probability of escape is for probability density functions i.e. density functions that integrate to 1, which constrains the results to be between 0 and 1, and hence results represent probabilities. Frequency of escape is for density functions i.e. density functions that integrate to more than 1, which means the results can be greater than 1, and hence results cannot be a probability. Event frequency of escape and annual frequency of escape is for either probability density functions or density functions.

4. The measures of hazard and risk using probability of hit/injury/death are:

   a. Probability of hit – P(H|R) – in hits per round;

   b. Frequency of hit – F(H|R) – in hits per round;

   c. Event individual risk of hit – IR(H|PE) – in hits per person per event;

   d. Event collective risk of hit – CR(H|E) – in hits per event;

   e. Annual individual risk of hit – IR(H|PY) – in hits per person per year;

   f. Annual collective risk of hit – CR(H|Y) – in hits per year.

5. Measures of hazard and risk using probability of hit/injury/death apply to density functions that represent probability of hit, or probability of injury, or probability of death. They can also be applied for combinations of these, for example, probability of injury or death. In the descriptions that follow we use the term probability of hit to cover any of these. These density functions can be used to calculate probability of hit, individual risk of hit and collective risk of hit.

6. Individual risk and collective risk can be annual or per event. Event individual risk uses the number of rounds fired per event whereas individual risk uses the number of rounds fired per year.

7. Collective risk calculations use the same data as individual risk calculations but also require population data, which can be provided indirectly as a population density or directly as the number of people in a given area. In the description that follows we use the term population density to cover either of these.

8. The calculations of hazard and risk are:

   a. Statistics (minimum, maximum, average and standard deviation) for quantities on a polygon;

   b. Statistics (minimum, maximum, average and standard deviation) for quantities inside and outside a polygon where outside extends to the edge of the TEA. Note that average values for polygons are obtained by scaling integrals by values related to the area of a polygon, for example the number of people within a polygon;

c. Integrals outside and inside a polygon where outside extends to the edge of the TEA.

D02. HAZARD AND RISK USING IMPACT DENSITY

9. Statistics on a polygon – ID–S. The statistics for impact density on a polygon are calculated.

a. Data – an impact probability density function, or an impact density function, and a polygon.

b. Primary results – the minimum, maximum, average and standard deviation of the impact density on the polygon.

10. Statistics inside and outside a polygon – ID–SIO. The statistics for impact density inside and outside a polygon are calculated.

a. Data – an impact probability density function, or an impact density function, and a polygon.

b. Results – the minimum, maximum, average and standard deviation of the impact density inside the polygon and the minimum, maximum, average and standard deviation of the impact density outside the polygon.

c. Method – integrals of the impact density over the polygon are calculated and scaled by the area of the polygon to obtain the average and standard deviation inside the polygon, and integrals of the impact density outside the polygon are calculated and scaled by the area of the TEA minus the area of the polygon to obtain the average and standard deviation outside the polygon.

11. Probability of escape – ID–P(E|R). The integrals of the impact densities outside and inside a polygon are calculated for a probability density.

a. Data – an impact probability density function and a polygon.

b. Results – the probability of escape for the polygon and the probability of containment for the polygon.

12. Frequency of escape – ID–F(E|R). The integrals of the impact densities outside and inside a polygon are calculated for a density function.

a. Data – an impact density function and a polygon.

b. Results – the frequency of escape for the polygon and the frequency of containment for the polygon.

13. Event frequency of escape – ID–F(E|E). The integrals of the impact densities outside and inside a polygon are calculated for either a probability density function or a density function and are scaled by the number of rounds fired per event.

a. Data – an impact probability density function or an impact density function, the number of rounds fired per event and a polygon.

b. Results – the event frequency of escape for the polygon and the event frequency of containment for the polygon.

14. Annual frequency of containment and escape – ID–F(E|Y). The integrals of the impact densities outside and inside a polygon are calculated for either a probability density function or a density function and are scaled by the number of rounds fired per year.

a. Data – an impact probability density function or an impact density function, the number of rounds fired per year and a polygon.

b. Results – the annual frequency of escape for the polygon and the annual frequency of containment for the polygon.

D03. HAZARD AND RISK USING PROBABILITY OR FREQUENCY OF HIT/INJURY/DEATH

1. Probability of hit statistics on a polygon – P(H)–S. The statistics for probability of hit on a polygon are calculated for rounds that would produce an impact probability density function.

a. Data – a probability of hit density function and a polygon.

b. Results – the minimum, maximum, average and standard deviation of the probability of hit on the polygon.

2. Frequency of hit statistics on a polygon – F(H)–S. The statistics for frequency of hit on a polygon are calculated for rounds that would produce a density function.

a. Data – a probability of hit density function and a polygon.

b. Results – the minimum, maximum, average and standard deviation the frequency of hit on the polygon.

3. Probability of hit statistics inside and outside a polygon – P(H)–SIO. The statistics for impact density inside and outside a polygon are calculated for rounds that would produce an impact probability density function.

a. Data – a probability of hit density function and a polygon.

b. Results – the minimum, maximum, average and standard deviation of the probability of hit inside the polygon and the minimum, maximum, average and standard deviation of the probability of hit outside the polygon.

c. Method – integrals of the probabilities of hit inside the polygon are calculated and scaled by the area of the polygon to get the average and standard deviation inside the polygon, and integrals of the probabilities of hit outside the polygon are calculated and scaled by the area of the TEA minus the area of the polygon to get the average and standard deviation outside the polygon.

4. Frequency of hit statistics inside and outside a polygon – F(H)–SIO. The statistics for frequency of hit inside and outside a polygon are calculated for rounds that would produce an impact density function.

a. Data – a probability of hit density function and a polygon.

b. Results – the minimum, maximum, average and standard deviation of the frequency of hit inside the polygon and the minimum, maximum, average and standard deviation of the frequency of hit outside the polygon.

c. Method – the integral of the frequencies of hit inside the polygon are calculated and scaled by the area of the polygon to get the average and standard deviation inside the polygon, and integrals of the probabilities of hit outside the polygon are calculated and scaled by the area of the TEA minus the area of the polygon to get the average and standard deviation outside.

5. Event individual risk of hit statistics on a polygon – IR(H|PE)–S. The statistics for individual risk of hit on a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per event, the population exposure and a polygon.

b. Results – the minimum, maximum, average and standard deviation for the individual risk of hit on the polygon.

6. Event individual risk of hit statistics inside and outside a polygon – IR(H|PE)–SIO. The statistics for individual risks of hit inside and outside a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per event, the population exposure and a polygon.

b. Results – the minimum, maximum, average and standard deviation for the individual risk of hit inside the polygon and the minimum, maximum, average and standard deviation for the individual risk of hit outside the polygon.

c. Method –integrals of the individual risks of hit inside the polygon are calculated and scaled by the area of the polygon to get the average and standard deviation inside, and integrals of the individual risk of hit outside the polygon are calculated and scaled by the area of the TEA minus the area of the polygon to get the average and standard deviation outside.

7. Event collective risk of hit – CR(H|E). The collective risks of hit inside and outside a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per event, the population exposure, the population density and a polygon.

b. Results – the collective risk of hit inside the polygon and the collective risk of hit outside the polygon.

c. Method – the integral of the individual risk of hit inside the polygon is calculated, and the integral of the individual risk of hit outside the polygon is calculated with both scaled by the population density and the number of rounds fired.

8. Annual individual risk of hit statistics on a polygon – IR(H|PY)–S. The statistics for individual risk of hit on a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per year, the population exposure and a polygon.

b. Results – the minimum, maximum, average and standard deviation for the individual risk of hit on the polygon.

9. Annual individual risk of hit statistics inside and outside a polygon – IR(H|PY)–SIO. The statistics for individual risks of hit inside and outside a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per year, the population exposure and a polygon.

b. Results – the minimum, maximum, average and standard deviation for the individual risk of hit inside the polygon and the minimum, maximum, average and standard deviation for the individual risk of hit outside the polygon.

c. Method –integrals of the individual risk of hit inside the polygon are calculated and scaled by the area of the polygon to get the average and standard deviation inside, and the integral of the individual risk of hit outside the polygon is calculated and scaled by the area of the TEA minus the area of the polygon to get the average and standard deviation outside.

10. Annual collective risk of hit – CR(H|Y). The collective risks of hit outside and inside a polygon are calculated.

a. Data – a probability of hit density function, the number of rounds fired per year, the population exposure, the population density and a polygon.

b. Results – the collective risk of hit inside the polygon and the collective risk of hit outside the polygon.

c. Method – the integral of the individual risk of hit inside the polygon is calculated, and the integral of the individual risk of hit outside the polygon is calculated with both scaled by the population density and the number of rounds fired.

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ANNEX E
OPTIONS FOR DEVELOPMENT OF WDA FROM DENSITY FUNCTIONS

E01. INTRODUCTION

1. This annex provides details of the options for the development of WDA from density functions in Chapter 7 using measures of hazard and risk in Chapter 2. For each of the combinations we list the data to be provided and the results obtained.

2. The measures of hazard and risk using impact density are:

   a. Impact density – ID – in impacts per square metre;

   b. Probability of escape – P(E|E) – in escapes per round;

   c. Frequency of escape – F(E|R) – in escapes per round;

   d. Event frequency of escape – F(E|E) – in escapes per event;

   e. Annual frequency of escape – F(E|Y) – in escapes per year;

3. Probability of escape is for probability density functions i.e. density functions that integrate to 1, which constrains the results to be between 0 and 1, and hence results represent probabilities. Frequency of escape is for density functions i.e. density functions that integrate to more than 1, which means the results can be greater than 1, and hence results cannot be a probability. Event frequency of escape and annual frequency of escape is for either probability density functions or density functions.

4. The measures of hazard and risk using probability of hit/injury/death are:

   a. Probability of hit – P(H|R) – in hits per round;

   b. Frequency of hit – F(H|R) – in hits per round;

   c. Event individual risk of hit – IR(H|E) – in hits per person per event;

   d. Event collective risk of hit – CR(H|E) – in hits per event;

   e. Annual individual risk of hit – IR(H|Y) – in hits per person per year;

   f. Annual collective risk of hit – CR(H|Y) – in hits per year.

5. Measures of hazard and risk using probability of hit/injury/death apply to density functions that represent probability of hit, or probability of injury, or probability of death. They can also be applied for combinations of these, for example, probability of injury or death. In the descriptions that follow we use the term probability of hit to cover any of these. These density functions can be used to calculate probability of hit, individual risk of hit and collective risk of hit.

6. Individual risk and collective risk can be annual or per event. Event individual risk uses the number of rounds fired per event whereas individual risk uses the number of rounds fired per year.

7. Collective risk calculations use the same data as individual risk calculations but also require population data, which can be provided indirectly as a population density or directly as the number of people in a given area. In the description that follows we use the term population density to cover either of these.

8. The methods for developing WDA are:

   a. Contours – C;

   b. Contour Hulls – CH – convex hulls around contours, and any additional points, with the contour having some required property;

c. Contour Danger Areas – CDA – specified shapes (such as that corresponding to a WDA developed using deterministic methodology) around contours, and any additional points, with the contour having some required property;

d. Hulls – H – convex hulls around contours, and any additional points, with the contours chosen so that the hull has some required property;

e. Danger Areas – DA – specified shapes around contours, and any additional points, with the contours chosen so that the danger area has some required property.

E02. WDA USING IMPACT DENSITY

1. When using impact density the meaning of integrals inside and outside the WDA depends on whether the impact density represents a probability or a frequency. For probability density functions i.e. a density function that integrates to 1, which constrains the results to be between 0 and 1, they represent a probability. For a density function i.e. a density function that integrates to more than 1, the results can be greater than 1, and hence cannot be a probability. The descriptions that follow cover both cases.

2. Impact density contours – ID-C. The contour corresponding to the specified impact density is found. The probability or frequency of escape and probability or frequency of containment for the contour is calculated.

a. Data – an impact probability density function or an impact density function, and an impact density value.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the probability or frequency of escape for the contour and the probability or frequency of containment for the contour.

3. Impact density contour hulls – ID-CH. The convex hull of the contour corresponding to the specified impact density, and any additional points, is found. The probability of escape and probability of containment for the convex hull are calculated.

a. Data – an impact probability density function or an impact density function, an impact density value and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the probability or frequency of escape for the contour hull and the probability or frequency of containment for the contour hull.

4. Impact density contour danger areas – ID-DA. A danger area containing the contour corresponding to the specified impact density, and any additional points, is found. The probability of escape and probability of containment for the danger area are calculated.

a. Data – an impact probability density function or an impact density function, an impact density value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the probability or frequency of escape for the danger area and the probability or frequency of containment for the danger area.

E03. WDA USING PROBABILITY OF ESCAPE

5. Probability of escape contours – P(E|R)-C. The contour in impact density corresponding to the specified probability of escape is found. The probability of containment for the contour is calculated.

a. Data – an impact probability density function and a probability of escape value.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the impact density value for the contour and the probability of containment for the contour.

6. Probability of escape contour hulls – P(E|R)-CH. The convex hull of the contour in impact density corresponding to the specified probability of escape, and any additional points, is found. The probability of escape and probability of containment for the convex hull are calculated.

a. Data – an impact probability density function, a probability of escape value and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the impact density value for the contour, the probability of escape for the contour hull and the probability of containment for the contour hull.

7. Probability of escape contour danger areas – P(E|R)-CDA. A danger area containing the contour in impact density corresponding to the specified probability of escape, and any additional points, is found. The probability of escape and probability of containment for the danger area are calculated.

a. Data – an impact probability density function, a probability of escape value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour, the probability of escape for the danger area and the probability of containment for the danger area.

8. Probability of escape hulls – P(E|R)-H. The convex hull of a contour in impact density, and any additional points, is found that corresponds to the specified probability of escape. The probability of containment for the hull is calculated.

a. Data – an impact probability density function, a probability of escape value, and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the impact density value for the danger area and the probability of containment for the hull.

d. Method – an impact density is found whose contour corresponds to the required probability of escape. The convex hull of the contour, and any additional points, is found and the probability of escape for this convex hull is calculated. The process is repeated with the impact density adjusted until the probability of escape for the hull is equal to that required.

9. Probability of escape danger areas – P(E|R)-DA. A danger area containing a contour in impact density, and any additional points, is found that corresponds to the specified probability of escape. The probability of containment for the danger area is calculated.

a. Data – an impact probability density function, a probability of escape value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour, the probability of escape for the danger area and the probability of containment for the danger area.

d. Method – an impact density is found whose contour corresponds to the required probability of escape. The contour and a danger area enclosing the contour, and any additional points, are found. The probability of escape for this danger area is calculated.

The process is repeated with the impact density adjusted until the probability of escape for the danger area is equal to that required.

E04. WDA USING FREQUENCY OF ESCAPE

1. Frequency of escape contours – F(E|R)-C. The contour in impact density corresponding to the specified frequency of escape is found. The frequency of containment for the contour is calculated.

a. Data – an impact density function and a frequency of escape value.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the impact density value for the contour and the frequency of containment for the contour.

2. Frequency of escape contour hulls – F(E|R)-CH. The convex hull of the contour in impact density corresponding to the specified frequency of escape, and any additional points, is found. The frequency of escape and frequency of containment for the convex hull are calculated.

a. Data – an impact density function, a frequency of escape value and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the impact density value for the contour, the frequency of escape for the contour hull and the frequency of containment for the contour hull.

3. Frequency of escape contour danger areas – P(E|R)-CDA. A danger area containing the contour in impact density corresponding to the specified frequency of escape, and any additional points, is found. The frequency of escape and frequency of containment for the danger area are calculated.

a. Data – an impact density function, a frequency of escape value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour, the frequency of escape for the danger area and the frequency of containment for the danger area.

4. Frequency of escape hulls – P(E|R)-H. The convex hull of a contour in impact density, and any additional points, is found that corresponds to the specified frequency of escape. The frequency of containment for the hull is calculated.

a. Data – an impact density function, a probability of escape value and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the impact density value for the contour and the frequency of containment for the hull.

d. Method – an impact density is found whose contour corresponds to the required frequency of escape. The convex hull of the contour, and any additional points, is found and the frequency of escape for this convex hull is calculated. The process is repeated with the impact density adjusted until the frequency of escape for the hull is equal to that required.

5. Frequency of escape danger areas – P(E|R)-DA. A danger area containing a contour in impact density, and any additional points, is found that corresponds to the specified frequency of escape. The frequency of containment for the danger area is calculated.

a. Data – an impact density function, a frequency of escape value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour and the frequency of containment for the danger area.

d. Method – an impact density is found whose contour corresponds to the required frequency of escape. The contour and a danger area enclosing the contour, and any additional points, are found. The frequency of escape for this danger area is calculated. The process is repeated with the impact density adjusted until the frequency of escape for the danger area is equal to that required.

E05. WDA USING EVENT FREQUENCY OF ESCAPE

1. Event frequency of escape contours – F(E|E)-C. The contour in impact density corresponding to the specified event frequency of escape is found. The event frequency of containment for the contour is calculated.

a. Data – an impact probability density function or an impact density function, an event frequency of escape value and the number of rounds fired per event.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the impact density value for the contour and the event frequency of containment for the contour.

2. Event frequency of escape contour hulls – F(E|E)-CH. The convex hull of the contour in impact density corresponding to the specified event frequency of escape, and any additional points, is found. The event frequency of escape and event frequency of containment for the convex hull are calculated.

a. Data – an impact probability density function or an impact density function, an event frequency of escape value, any optional additional points and the number of rounds fired per event.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the impact density value for the contour, the event frequency of escape for the contour hull and the event frequency of containment for the contour hull.

3. Event frequency of escape contour danger areas – P(E|E)-CDA. A danger area containing the contour in impact density corresponding to the specified event frequency of escape, and any additional points, is found. The event frequency of escape and event frequency of containment for the danger area are calculated.

a. Data – an impact probability density function or an impact density function, an event frequency of escape value, any optional additional points and the number of rounds fired per event.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour, the event frequency of escape for the danger area and the event frequency of containment for the danger area.

4. Event frequency of escape hulls – P(E|E)-H. The convex hull of a contour in impact density, and any additional points, is found that corresponds to the specified event frequency of escape. The event frequency of containment for the hull is calculated.

a. Data – an impact probability density function or an impact density function, an event frequency of escape value, any optional additional points and the number of rounds fired per event.

b. Primary result – the polygon representing the hull.

c. Secondary results – the impact density value for the contour and the event frequency of containment for the hull.

d. Method – an impact density is found whose contour corresponds to the required event frequency of escape. The convex hull of the contour, and any additional points, is found and the event frequency of escape for this convex hull is calculated. The process is repeated with the impact density adjusted until the event frequency of escape for the hull is equal to that required.

5. Event frequency of escape danger areas – P(E|E)-DA. A danger area containing a contour in impact density, and any additional points, is found that corresponds to the specified event frequency of escape. The event frequency of containment for the danger area is calculated.

a. Data – an impact density function, an event frequency of escape value, any optional additional points and the number of rounds fired per event.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour and the event frequency of containment for the danger area.

d. Method – an impact density is found whose contour corresponds to the required event frequency of escape. The contour and a danger area enclosing the contour, and any additional points, are found. The event frequency of escape for this danger area is calculated. The process is repeated with the impact density adjusted until the event frequency of escape for the danger area is equal to that required.

E06. WDA USING ANNUAL FREQUENCY OF ESCAPE

6. Annual frequency of escape contours – F(E|Y)-C. The contour in impact density corresponding to the specified annual frequency of escape is found. The annual frequency of containment for the contour is calculated.

a. Data – an impact probability density function or an impact density function, an annual frequency of escape value and the number of rounds fired per year.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the impact density value for the contour and the annual frequency of containment for the contour.

7. Annual frequency of escape contour hulls – F(E|Y)-CH. The convex hull of the contour in impact density corresponding to the specified annual frequency of escape, and any additional points, is found. The annual frequency of escape and annual frequency of containment for the convex hull are calculated.

a. Data – an impact probability density function or an impact density function, an annual frequency of escape value, any optional additional points and the number of rounds fired per year.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the impact density value for the contour, the annual frequency of escape for the contour hull and the annual frequency of containment for the contour hull.

8. Annual frequency of escape contour danger areas – P(E|Y)-CDA. A danger area containing the contour in impact density corresponding to the specified annual frequency of escape, and any additional points, is found. The annual frequency of escape and annual frequency of containment for the danger area are calculated.

a. Data – an impact probability density function or an impact density function, an annual frequency of escape value, any optional additional points and the number of rounds fired per year.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value, the annual frequency of escape for the danger area and the annual frequency of containment for the danger area.

9. Annual frequency of escape hulls – P(E|Y)-H. The convex hull of a contour in impact density, and any additional points, is found that corresponds to the specified annual frequency of escape. The annual frequency of containment for the hull is calculated.

a. Data – an impact probability density function or an impact density function, an annual frequency of escape value, any optional additional points and the number of rounds fired per year.

b. Primary result – the polygon representing the hull.

c. Secondary results – the impact density value for the contour and the annual frequency of containment for the hull.

d. Method – an impact density is found whose contour corresponds to the required annual frequency of escape. The convex hull of the contour, and any additional points, is found and the annual frequency of escape for this convex hull is calculated. The process is repeated with the impact density adjusted until the annual frequency of escape for the hull is equal to that required.

10. Annual frequency of escape danger areas – P(E|Y)-DA. A danger area containing a contour in impact density, and any additional points, is found that corresponds to the specified annual frequency of escape. The annual frequency of containment for the danger area is calculated.

a. Data – an impact probability density function or an impact density function, an annual frequency of escape value, any optional additional points and the number of rounds fired per year.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the impact density value for the contour and the annual frequency of containment for the danger area.

d. Method – an impact density is found whose contour corresponds to the required annual frequency of escape. The contour and a danger area enclosing the contour, and any additional points, are found. The annual frequency of escape for this danger area is calculated. The process is repeated with the impact density adjusted until the annual frequency of escape for the danger area is equal to that required.

E07. WDA USING PROBABILITY OF HIT/INJURY/DEATH

1. Probability of hit contours – P(H|R)-C. The contour corresponding to the specified probability of hit is found. The average probability of hit inside and the average probability of hit outside the contour are calculated.

   a. Data – a probability of hit density function and a probability of hit value.

   b. Primary result – the polygon(s) representing the contour.

   c. Secondary results – the average probability of hit inside the contour and the average probability of hit outside the contour.

2. Probability of hit contour hulls – P(H|R)-CH. The convex hull of the contour corresponding to the specified probability of hit, and any additional points, is found. The average probability of hit inside and the average probability of hit outside the convex hull are calculated.

   a. Data – a probability of hit density function, a probability of hit value and any optional additional points.

   b. Primary result – the polygon representing the contour hull.

   c. Secondary results – the average probability of hit inside the contour hull and the average probability of hit outside the contour hull.

3. Probability of hit contour danger areas – P(H|R)-CDA. A danger area containing the contour corresponding to a specified probability of hit, and any additional points, is found. The average probability of hit inside and the average probability of hit outside the danger area are calculated.

   a. Data – a probability of hit density function, a probability of hit value and any optional additional points.

   b. Primary result – the polygon representing the danger area.

   d. Secondary results – the average probability of hit inside the danger area and the average probability of hit outside the danger area.

4. Probability of hit hulls – P(H|R)-H. The convex hull of a contour in probability of hit, and any additional points, is found that corresponds to a specified average probability of hit outside the hull. The average probability of hit inside the hull is calculated.

   a. Data – a probability of hit density function, an average probability of hit value and any optional additional points.

   b. Primary result – the polygon representing the hull.

   c. Secondary results – the probability of hit value for the contour and the average probability of hit inside the hull.

   d. Method – the contour at the required probability of hit and the convex hull of the contour, and any additional points, are found. The average probability of hit outside this hull is calculated. The process is repeated with the probability of hit for the contour adjusted until the average probability of hit outside the hull is equal to that required.

5. Probability of hit danger areas – P(H|R)-DA. A danger area containing a contour in probability of hit, and any additional points, is found that corresponds to a specified average probability of hit. The average probability of hit outside and average probability of hit inside the danger area are calculated.

   a. Data – a probability of hit density function, an average probability of hit value and any optional additional points.

   b. Primary result – the polygon representing the danger area.

c. Secondary results – the probability of hit value for the contour and the average probability of hit inside the danger area.

d. Method – the contour at the required probability of hit and a danger area enclosing the contour, and any additional points, are found. The average probability of hit outside this danger area is calculated. The process is repeated with the probability of hit for the contour adjusted until the average probability of hit outside the danger area is equal to that required.

E08. WDA USING FREQUENCY OF HIT/INJURY/DEATH

6. Frequency of hit contours – F(H|R)-C. The contour corresponding to the specified frequency of hit is found. The average frequency of hit inside and the average frequency of hit outside the contour are calculated.

a. Data – a probability of hit density function and a frequency of hit value.

b. Primary result – the polygon(s) representing the contour.

c. Secondary results – the average probability of hit inside the contour and the average probability of hit outside the contour.

7. Frequency of hit contour hulls – F(H|R)-CH. The convex hull of the contour corresponding to the specified frequency of hit, and any additional points, is found. The average frequency of hit inside and the average frequency of hit outside the convex hull are calculated.

a. Data – a probability of hit density function, a frequency of hit value and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the average frequency of hit inside the contour hull and the average frequency of hit outside the contour hull.

8. Frequency of hit contour danger areas – F(H|R)-CDA. A danger area containing the contour corresponding to a specified frequency of hit, and any additional points, is found. The average frequency of hit inside and the average frequency of hit outside the danger area are calculated.

a. Data – a probability of hit density function, a frequency of hit value and any optional additional points.

b. Primary result – the polygon representing the danger area.

d. Secondary results – the average frequency of hit inside the danger area and the average frequency of hit outside the danger area.

9. Frequency of hit hulls – F(H|R)-H. The convex hull of a contour in probability of hit, and any additional points, is found that corresponds to a specified average frequency of hit outside the hull. The average frequency of hit inside the hull is calculated.

a. Data – a probability of hit density function, an average probability of hit value and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the frequency of hit value for the contour and the average frequency of hit inside the hull.

d. Method – the contour at the required frequency of hit and the convex hull of the contour, and any additional points, are found. The average frequency of hit outside this hull is calculated. The process is repeated with the frequency of hit for the contour adjusted until the average frequency of hit outside the hull is equal to that required.

10. Frequency of hit danger areas – F(H|R)-DA. A danger area containing a contour in frequency of hit, and any additional points, is found that corresponds to a specified average frequency of hit. The average frequency of hit outside and average frequency of hit inside the danger area are calculated.

a. Data – a probability of hit density function, an average frequency of hit value and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the frequency of hit value for the contour and the average frequency of hit inside the danger area.

d. Method – the contour at the required frequency of hit and a danger area enclosing the contour, and any additional points, are found. The average frequency of hit outside this danger area is calculated. The process is repeated with the frequency of hit for the contour adjusted until the average frequency of hit outside the danger area is equal to that required.

E09. WDA USING EVENT INDIVIDUAL RISK OF HIT/INJURY/DEATH

1. Event individual risk contours – IR(H|E)-C. The contour corresponding to the specified event individual risk of hit is found. The average event individual risk of hit inside and the average event individual risk of hit outside the contour are calculated.

a. Data – a probability of hit density function, an event individual risk of hit value, the number of rounds fired per event and the population exposure.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the average event individual risk of hit inside the contour and the average event individual risk of hit outside the contour.

2. Event individual risk contour hulls – IR(H|E)-CH. The convex hull of the contour corresponding to the specified event individual risk of hit, and any additional points, is found. The average event individual risk of hit inside and the average event individual risk of hit outside the convex hull are calculated.

a. Data – a probability of hit density function, an event individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the average event individual risk of hit inside the contour hull and the average event individual risk of hit outside the contour hull.

3. Event individual risk contour danger areas – IR(H|E)-CDA. A danger area containing the contour corresponding to the specified event individual risk of hit, and any additional points, is found. The average event individual risk of hit inside and the average event individual risk of hit outside the danger area are calculated.

a. Data – a probability of hit density function, an event individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the average event individual risk of hit inside the danger area and the average event individual risk of hit outside the danger area.

4. Event individual risk hulls – IR(H|E)-H. The convex hull of a contour in event individual risk of hit, and any additional points, is found that corresponds to the specified average event individual risk of hit outside the hull. The average event individual risk of hit inside the hull is calculated.

a. Data – a probability of hit density function, an average event individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the event individual risk of hit value for the contour and the average event individual risk of hit inside the hull.

d. Method – the contour at the required event individual risk of hit and the convex hull of the contour, and any additional points, are found. The average event individual risk of hit outside this hull is calculated. The process is repeated with the event individual risk of hit for the contour adjusted until the average event individual risk of hit outside the hull is equal to that required.

5. Event individual risk danger areas – IR(H|E)-DA. A danger area containing a contour in event individual risk of hit, and any additional points, is found that corresponds to the specified average event individual risk of hit outside the danger area. The average event individual risk of hit inside the danger area is calculated.

a. Data – a probability of hit density function, an average event individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the danger area

c. Secondary results – the event individual risk of hit value for the contour and the average event individual risk of hit inside the danger area.

d. Method – the contour at the required event individual risk of hit and a danger area enclosing the contour, and any additional points, are found. The average event individual risk of hit outside this danger area is calculated. The process is repeated with the event individual risk of hit for the contour adjusted until the average event individual risk of hit outside the danger area is equal to that required.

E10. WDA USING EVENT COLLECTIVE RISK OF HIT/INJURY/DEATH

1. Event collective risk contours – CR(H)-C. The contour in individual risk of hit corresponding to the specified event collective risk of hit outside the contour is found. The event collective risk of hit inside the contour is calculated.

a. Data – a probability of hit density function, an event collective risk of hit value, the number of rounds fired per year, the population exposure and the population density.

b. Primary result – the polygon(s) representing the contour.

c. Secondary results – the individual risk value for the contour and the collective risk of hit inside the contour.

2. Event collective risk contour hulls – CR(H)-CH. The convex hull of the contour in individual risk of hit corresponding to the specified event collective risk hit outside the contour, and any additional points, is found. The collective risk of hit inside the contour is calculated. The collective risks of hit inside and outside the convex hull are calculated.

a. Data – a probability of hit density function, a collective risk of hit value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the individual risk value for the contour, the collective risk of hit inside the contour, and the collective risk of hit inside the contour hull and the collective risk of hit outside the contour hull.

3. Event collective risk hulls – CR(H)-H. The convex hull of a contour in individual risk of hit, and any additional points, is found that corresponds to the specified event collective risk of hit outside the hull. The collective risks of hit inside and outside the contour are calculated. The collective risk of hit inside the hull is calculated.

a. Data – a probability of hit density function, a collective risk of hit outside the hull value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the individual risk of hit value for the contour, the collective risk of hit inside the contour, the collective risk of hit outside the contour, and the collective risk of hit inside the hull.

d. Method – the contour at the required individual risk of hit and the convex hull of the contour, and any additional points, are found. The average individual risk of hit outside this convex hull is calculated. The individual risk of hit for the contour is adjusted until the average individual risk of hit outside the hull is equal to the required value.

4. Event collective risk danger areas – CR(H)-CDA. A danger area containing the contour in individual risk corresponding to the specified event collective risk of hit outside the contour, and any additional points, is found. The collective risk of hit inside the contour is calculated. The collective risks of hit inside and outside the danger area are calculated.

a. Data – a probability of hit density function, an individual risk of hit value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the danger area.

d. Secondary results – the individual risk value for the contour, the collective risk of hit inside the contour, the collective risk of hit inside the danger area and the collective risk of hit outside the danger area.

5. Event collective risk danger areas – CR(H)-DA. A danger area containing a contour in individual risk of hit, and any additional points, is found that corresponds to the specified event collective risk of hit outside the danger area. The collective risks of hit inside and outside the contour are calculated. The collective risk of hit inside the danger area is calculated.

a. Data – a probability of hit density function, a collective risk of hit outside the danger area value, the number of rounds fired per year, the population density or the number of people and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the individual risk of hit value for the contour, the collective risk of hit inside the contour, the collective risk of hit outside the contour and the collective risk of hit inside the danger area.

d. Method – the contour at the required individual risk of hit and a danger area enclosing the contour, and any additional points, are found. The average individual risk of hit outside this danger area is calculated. The individual risk of hit for the contour is adjusted until the average individual risk of hit outside the danger area is equal to the required value.

E11. WDA USING ANNUAL INDIVIDUAL RISK OF HIT/INJURY/DEATH

6. Annual individual risk contours – IR(H|Y)-C. The contour corresponding to the specified annual individual risk of hit is found. The average annual individual risk of hit inside and the average annual individual risk of hit outside the contour are calculated.

a. Data – a probability of hit density function, an annual individual risk of hit value, the number of rounds fired per event and the population exposure.

b. Primary result – the polygon(s) representing contour.

c. Secondary results – the average annual individual risk of hit inside the contour and the average annual individual risk of hit outside the contour.

7. Annual individual risk contour hulls – IR(H|Y)-CH. The convex hull of the contour corresponding to the specified annual individual risk of hit, and any additional points, is found. The average annual individual risk of hit inside and the average annual individual risk of hit outside the convex hull are calculated.

a. Data – a probability of hit density function, an annual individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the average annual individual risk of hit inside the contour hull and the average annual individual risk of hit outside the contour hull.

8. Annual individual risk contour danger areas – IR(H|Y)-CDA. A danger area containing the contour corresponding to the specified annual individual risk of hit, and any additional points, is found. The average annual individual risk of hit inside and the average annual individual risk of hit outside the danger area are calculated.

a. Data – a probability of hit density function, an annual individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the average annual individual risk of hit inside the danger area and the average annual individual risk of hit outside the danger area.

9. Annual individual risk hulls – IR(H|Y)-H. The convex hull of a contour in annual individual risk of hit, and any additional points, is found that corresponds to the specified average annual individual risk of hit outside the hull. The average annual individual risk of hit inside the hull is calculated.

a. Data – a probability of hit density function, an average annual individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the annual individual risk of hit value for the contour and the average annual individual risk of hit inside the hull.

d. Method – the contour at the required annual individual risk of hit and the convex hull of the contour, and any additional points, are found. The average individual risk of hit outside this hull is calculated. The process is repeated with the annual individual risk of hit for the contour adjusted until the average annual individual risk of hit outside the hull is equal to that required.

10. Annual individual risk danger areas – IR(H|Y)-DA. A danger area containing a contour in annual individual risk of hit, and any additional points, is found that corresponds to the specified average

annual individual risk of hit outside the danger area. The average annual individual risk of hit inside the danger area is calculated.

a. Data – a probability of hit density function, an average annual individual risk of hit value, the number of rounds fired per event, the population exposure and any optional additional points.

b. Primary result – the polygon representing the danger area

c. Secondary results – the annual individual risk of hit value for the contour and the average annual individual risk of hit inside the danger area.

d. Method – the contour at the required annual individual risk of hit and a danger area enclosing the contour, and any additional points, are found. The average annual individual risk of hit outside this danger area is calculated. The process is repeated with the annual individual risk of hit for the contour adjusted until the average annual individual risk of hit outside the danger area is equal to that required.

WDA USING ANNUAL COLLECTIVE RISK OF HIT/INJURY/DEATH

1. Annual collective risk contours – CR(H)-C. The contour in individual risk of hit corresponding to the specified annual collective risk of hit outside the contour is found. The annual collective risk of hit inside the contour is calculated.

a. Data – a probability of hit density function, an annual collective risk of hit value, the number of rounds fired per year, the population exposure and the population density.

b. Primary result – the polygon(s) representing the contour.

c. Secondary results – the individual risk value for the contour and the annual collective risk of hit inside the contour.

2. Annual collective risk contour hulls – CR(H)-CH. The convex hull of the contour in individual risk of hit corresponding to the specified annual collective risk hit outside the contour, and any additional points, is found. The collective risk of hit inside the contour is calculated. The collective risks of hit inside and outside the convex hull are calculated.

a. Data – a probability of hit density function, an annual collective risk of hit value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the contour hull.

c. Secondary results – the individual risk value for the contour, the annual collective risk of hit inside the contour, the annual collective risk of hit inside the contour hull and annual collective risk of hit outside the contour hull.

3. Annual collective risk danger areas – CR(H)-CDA. A danger area containing the contour in individual risk corresponding to the specified annual collective risk of hit outside the contour, and any additional points, is found. The annual collective risk of hit inside the contour is calculated. The annual collective risks of hit inside and outside the danger area are calculated.

a. Data – a probability of hit density function, an annual collective risk of hit value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the danger area.

d. Secondary results – the individual risk value for the contour, the collective risk of hit inside the contour, the collective risk of hit inside the danger area and the collective risk of hit outside the danger area.

4. Annual collective risk hulls – CR(H)-H. The convex hull of a contour in individual risk of hit, and any additional points, is found that corresponds to the specified annual collective risk of hit outside the hull. The annual collective risks of hit inside and outside the contour are calculated. The annual collective risk of hit inside the hull is calculated.

a. Data – a probability of hit density function, an annual collective risk of hit outside the hull value, the number of rounds fired per year, the population exposure, the population density and any optional additional points.

b. Primary result – the polygon representing the hull.

c. Secondary results – the individual risk of hit value for the contour, the collective risk of hit inside, the collective risk of hit outside the contour and the collective risk of hit inside the hull.

d. Method – the contour at the required individual risk of hit and the convex hull of the contour, and any additional points, are found. The annual collective risk of hit outside this hull is calculated. The individual risk of hit for the contour is adjusted until the average annual collective risk of hit outside the hull is equal to the required value.

5. Annual collective risk danger areas – CR(H)-DA. A danger area containing a contour in individual risk of hit, and any additional points, is found that corresponds to the specified annual collective risk of hit outside the danger area. The annual collective risks of hit inside and outside the contour are calculated. The annual collective risk of hit inside the danger area is calculated.

a. Data – a probability of hit density function, an annual collective risk of hit outside the danger area value, the number of rounds fired per year, the population density or the number of people and any optional additional points.

b. Primary result – the polygon representing the danger area.

c. Secondary results – the individual risk of hit value for the contour, the annual collective risk of hit inside contour, the annual collective risk of hit outside the contour and the annual collective risk of hit inside the danger area.

d. Method – the contour at the required annual collective risk of hit and a danger area enclosing the contour, and any additional points, are found. The collective risk of hit outside this danger area is calculated. The collective risk of hit for the contour is adjusted until the collective risk of hit outside the danger area is equal to the required value.

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Air Danger height (ADH)
The maximum height above ground level at which hazards may exist.
Notes:
1. The air danger height is measured in feet above ground level.
2. Altitude is measured above mean sea level.
[Derived from IRSAG Glossary of Terms (Reference 29)]

Acceptable risk
A level of *risk*, associated with some activity, that is accepted without detailed demonstration of the benefits from the activity.
Notes:
Related term: tolerable risk.
[Derived from ISO/IEC Guide 51]

Aleatory uncertainty
variability (admitted)
Lack of certainty arising from, or associated with, the inherent natural randomness of a system or process.

Burst safety distance (BSD)
Hazard distance, calculated for still air at sea level, away from a fragmenting weapon.
Related terms: normal burst safety distance; reduced burst safety distance.

Collective risk (CR) of death
Collective risk (admitted)
The risk to an exposed population.
Notes:
1. Annual collective risk has units of deaths per year.
2. Event collective risk has units of deaths per event
3. Collective risk can be calculated for other consequences such as hit or injury.

Consequence
Outcome of an event.
Notes:
1. There can be more than one consequence from one event.
2. Consequences can range from positive to negative. However, consequences are always negative for safety aspects.
3. Consequences can be expressed qualitatively or quantitatively.
[ISO/IEC Guide 73]

Data cube
Data structure and methodology to enable look up of distribution parameters.
Notes:
1. Data cubes usually contain statistical data for weapon accuracy, ricochet and weapon failure modes.

Epistemic uncertainty
uncertainty (admitted)
Lack of certainty associated with a model of a system or process that arises from limitations on causal understanding.

Event
Occurrence of a particular set of circumstances.
Notes:
1. The event can be certain or uncertain.
2. The event can be a single occurrence or a series of occurrences.

3. The *probability* associated with the event can be estimated for a given period of time.
[ISO/IEC Guide 73]

Explosive ordnance disposal (EOD)
The detection, identification, on-site evaluation, rendering safe, recovery and final disposal of unexploded explosives ordnance. It may also include explosives ordnance which has become hazardous by damage or deterioration.
[AAP-6 (Reference 28)]

Frequency
The number of occurrences of a given event or the number of observations falling into a specified class.
[Derived from ISO 3534-1]

Harm
Physical injury or damage to the health of people, or damage to property or the environment.
[ISO/IEC Guide 51]

Hazard
Potential source of *harm*.
Note: The term hazard can be qualified in order to define its origin or the nature of the expected harm (e.g. electric shock hazard, crushing hazard, cutting hazard, toxic hazard, fire hazard, drowning hazard).
[ISO/IEC Guide 51]

Hazard identification
Process to find, list and characterize *hazards*.[Derived from ISO/IEC Guide 73]

Importance sampling
A method of random sampling that involves sampling uniformly from the range of a distribution and weighting the samples with the probability density function calculated at the sampled point.
Note: Simulations that use this method of sampling are more efficient than those that use *Monte Carlo simulation* but do not directly emulate the real problem.

Individual risk (IR) of death
Individual risk (admitted)
The risk to a single person.
Notes:
1. Annual individual risk has units of deaths per person per year.
2. Event individual risk has units of deaths per person per event.
3. Inidividual risk can be calculated for other consequences such as hit or injury.
[Derived from AOP-38 (Reference 29)]

Monte Carlo simulation
A method utilising random sampling to obtain inputs for computer simulation trials and obtaining approximate solutions in terms of a range of values each of which has a calculated probability of being the solution to the problem.
[ARMP-07]

Normal burst safety distance (NBSD)
The distance from the point on the ground, at or below the point of burst, beyond which it is improbable that any fragment from a bursting weapon will travel.
[IRSAG GoT]

One sided confidence interval
Confidence limit (admitted)

The interval from the smallest possible value of θ up to T (or the interval from T up to the largest possible value of θ) is a one-sided $(1-\alpha)$ confidence interval for θ when T is a function of the observed values such that, θ being a population parameter to be estimated, the probability $P(T \geq \theta)$ [or the probability $P(T \leq \theta)$] is at least equal to $(1-\alpha)$ [where $(1-\alpha)$ is a fixed number, positive and less than 1].

Notes:

1. The limit T of the confidence interval is a statistic and as such will generally assume different values from sample to sample.
2. In a long series of samples, the relative frequency of cases where the true value of the population parameter θ is covered by the confidence interval is greater than or equal to $(1-\alpha)$.

Related term: two sided confidence interval.
[Derived from ISO 3534-1]

p fractile
p fractile of a random variable (admitted)
p fractile of a probability distribution (admitted)

The value of the random variable for which the distribution function equals p $(0 < p < 1)$ or “jumps” from a value less than p to a value greater than p.

Notes:

1. If the distribution function equals p throughout an interval between two consecutive values of the random variable, then any value in this interval may be considered as the p fractile.
2. x_p is the p-fractile if

$$P_r\left(X < x_p\right) \leq p \leq P_r\left(X \leq x_p\right)$$

3. In the case of a continuous variable, the p fractile is a value of the variable below which the proportion p of the distribution lies.

[Derived from ISO 3534-1]

Probability

A real number in the scale 0 to 1 attached to a random event.
Note: It can be related to a long-run relative frequency of occurrence or to a degree of belief that an event will occur. For a high degree of belief, the probability is near 1.
[ISO 3534-1]

Probability density function

The derivative (when it exists) of the distribution function for a continuous random variable

$$f(x) = \frac{dF(x)}{dx}.$$

Note: $f(x)dx$ is the “probability element”

$$f(x)dx = P_r\left(x < X < x + dx\right).$$

[ISO 3534-1]

Probability of escape

The *probability* that a hazardous projectile escapes from a prescribed weapon danger area.

Reduced burst safety distance RBSD
The distance from the point on the ground, at or below the point of burst, beyond which it is improbable that more than one fragment per bursting weapon could travel.
[IRSAG GoT]

Risk
Combination of the *probability* of occurrence of *harm* and the severity of that *harm*.
Note: Frequency rather than probability may be used in describing risk.
[ISO/IEC Guide 51]

Risk analysis
Systematic use of available information to identify *hazards* and to estimate the *risk*.
[ISO/IEC Guide 51]

Risk assessment
Overall process comprising a *risk analysis* and a *risk evaluation*.
[AOP-38 and ISO/IEC Guide 51]

Risk estimation
Process used to assign values to the probability and consequences of a risk.
[ISO/IEC Guide 73]

Risk evaluation
Procedure based on the *risk analysis* to determine whether the *tolerable risk* has been achieved.
[ISO/IEC Guide 51]

Risk management
The systematic application of management policies, procedures and practices to the tasks of analysing, evaluating and treating risks.
[Derived from AOP-38]

Risk treatment
Process of selection and implementation of measures to modify *risk*.
Notes:
1. The term is sometimes used for the measures themselves.
2. Risk treatment measures can include avoiding, optimizing, transferring or retaining risk.
[ISO/IEC Guide 73]

Safety
An acceptable level of freedom from *risks* to personnel and material at all times recognizing the considerations of operational necessity as a limiting factor.
[Derived from AOP-38]

Sensitivity
The rate of change of a function $f\left(x_1, x_2, \ldots, x_{n-1}, x_n\right)$ with respect to one of its input variables:

$$S_{x_i}\left(X_i\right) = \frac{\partial f}{\partial x_i}\left(x_1, x_2, \ldots, x_i = X_i, \ldots, x_{n-1}, x_n\right).$$

Note: Sensitivity is usually calculated for a function that represents the solution to a particular problem.
Example: The sensitivity of a weapon danger area with respect to wind speed is estimated by developing danger areas for two different wind speeds and calculating the change in size between these divided by the difference in the two different wind speeds.

Smearing
An expansion of a distribution to cover uncertainties in quantities such as launch location or heading that have not been explicitly modelled.

Standard deviation
The positive square root of the *variance* of a random variable or of a probability distribution

$$\sigma = \sqrt{V(X)}$$

[ISO 3534-1]

Submunition
Any munition that, to perform its task, separates from a parent munition.
[NA, AAP-6]

Tolerable risk
A level of *risk*, higher than *acceptable risk*, associated with some activity, that is tolerated through *risk evaluation* so as to secure certain benefits from the activity.
Note: In developing *weapon danger areas* for training *acceptable risk* is used.

Total energy area (TEA)
The maximum possible two dimensional space around a firing point within which all weapon system effects are contained.

Total energy zone (TEZ)
The maximum possible three dimensional space around a firing point within which all weapon system effects are contained.

Two sided confidence interval
confidence interval (admitted)
The interval between T_1 and T_2 is a two-sided $(1-\alpha)$ confidence interval for θ when T_1 and T_2 are two functions of the observed values such that, θ being a population parameter to be estimated, the probability $P(T_1 \le \theta \le T_2)$ is at least equal to $(1-\alpha)$ [where $(1-\alpha)$ is a fixed number, positive and less than 1].
Notes:
1. The limits T_1 and T_2 of the confidence interval are statistics and as such will generally assume different values from sample to sample.
2. In a long series of samples, the relative frequency of cases where the true value of the population parameter θ is covered by the confidence interval is greater than or equal to $(1-\alpha)$.

Related term: one sided confidence interval.
[Derived from ISO 3534-1]

Variance
The expectation of the square of the centred random variable

$$\sigma^2 = V(X) = E\left[X - E(X)\right]^2$$

Note: The variance is a measure of the spread of a variable about the mean.
Related term: standard deviation
[Derived from ISO 3534-1]

Weapon danger area (WDA)
The area associated with firing a weapon where the risk of death or injury exceeds some threshold.
Notes:
(1) The risk outside the weapon danger area does not exceed this threshold and hence the risk to people outside the weapon danger area is acceptable or tolerable.
(2) A weapon danger area is determined for normal firing conditions and excludes, for example, deliberate misfiring.
(3) Because risk thresholds are not zero and normal firing conditions are used weapon danger areas are smaller than the *total energy area*.
Related term: weapon danger zone.

Weapon danger zone (WDZ)
The three dimensional space associated with firing a weapon where the risk of death or injury exceeds some threshold.
Notes:
(1) The risk outside the weapon danger zone does not exceed this threshold and hence the risk to people outside the weapon danger zone is acceptable or tolerable.
(2) A weapon danger zone is determined for normal firing conditions and excludes, for example, deliberate misfiring.
(3 Because risk thresholds are not zero and normal firing conditions are used weapon danger zones are smaller than the *total energy zone*.
Related term: weapon danger area.

Weapon danger area template
A technical drawing of a *weapon danger area* for a single weapon and a single target, projected on a specified line of fire, worked to a given scale and produced on appropriate material for convenient application to a map.
[Derived from IRSAG Glossary of Terms]

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